



Edexcel AS Mathematics

8MA0/01 - Pure Mathematics

Friday 20 May 2022

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2022 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Find $\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5 \right) dx$, giving your answer in simplest form. (4)

Model solution

Write the square-root term as a power of x , then integrate each term using the power rule.

$$\int \left(8x^3 - \frac{3}{2}x^{-1/2} + 5 \right) dx = \frac{8x^4}{4} - \frac{3}{2} \frac{x^{1/2}}{1/2} + 5x + C.$$

$$2x^4 - 3\sqrt{x} + 5x + C$$

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Question 2

Question 2

$$f(x) = 2x^3 + 5x^2 + 2x + 15.$$

- (a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$. (2)
- (b) Find the constants a, b, c such that $f(x) = (x + 3)(ax^2 + bx + c)$. (2)
- (c) Hence show that $f(x) = 0$ has only one real root. (2)
- (d) Write down the real root of $f(x - 5) = 0$. (1)

Model solution

(a) The factor theorem requires evaluation at $x = -3$.

$$f(-3) = 2(-3)^3 + 5(-3)^2 + 2(-3) + 15 = -54 + 45 - 6 + 15 = 0.$$

Since $f(-3) = 0$, $(x + 3)$ is a factor of $f(x)$.

(b) Expand the proposed factorisation and compare coefficients.

$$(x + 3)(ax^2 + bx + c) = ax^3 + (b + 3a)x^2 + (c + 3b)x + 3c.$$

Thus $a = 2$, $b + 6 = 5$ and $3c = 15$, giving

$$a = 2, \quad b = -1, \quad c = 5, \text{ so } f(x) = (x + 3)(2x^2 - x + 5).$$

(c) The discriminant of the quadratic factor is

$$(-1)^2 - 4(2)(5) = -39 < 0.$$

The quadratic $2x^2 - x + 5 = 0$ therefore has no real roots. The linear factor gives just $x = -3$.

$f(x) = 0$ has exactly one real root, $x = -3$.

(d) The argument of f must equal its only real root: $x - 5 = -3$.

$$x = 2$$

Question 3

Question 3

In triangle PQR, $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$.

(a) Find $\overrightarrow{QR}$. (2)

(b) Hence find $|\overrightarrow{QR}|$ as a simplified surd. (2)

The point S lies on the line segment QR with $QS : SR = 3 : 2$.

(c) Find $\overrightarrow{PS}$. (2)

Model solution

(a) Travel from Q to P, then from P to R.

$$\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ} = (13\mathbf{i} - 15\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j}).$$

$$\overrightarrow{QR} = 10\mathbf{i} - 20\mathbf{j}$$

(b) Use the magnitude formula.

$$|\overrightarrow{QR}| = \sqrt{10^2 + (-20)^2} = \sqrt{500}.$$

$$|\overrightarrow{QR}| = 10\sqrt{5}$$

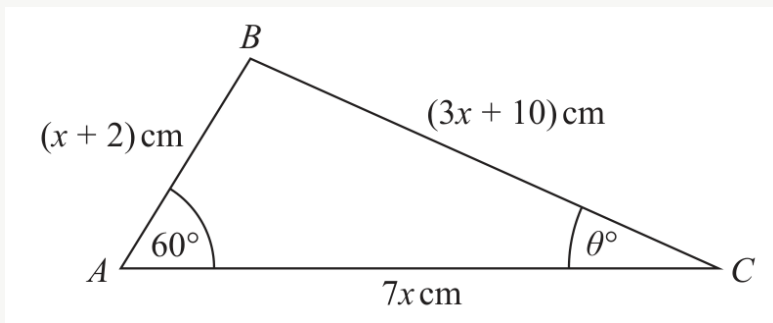
(c) Since S is three fifths of the way from Q to R, $\overrightarrow{QS} = \frac{3}{5}\overrightarrow{QR}$.

$$\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS} = 3\mathbf{i} + 5\mathbf{j} + \frac{3}{5}(10\mathbf{i} - 20\mathbf{j}).$$

$$\overrightarrow{PS} = 9\mathbf{i} - 7\mathbf{j}$$

Question 4

Question 4



Triangle ABC has $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, $\angle BAC = 60^\circ$ and $\angle ACB = \theta^\circ$.

(a)(i) Show that $17x^2 - 35x - 48 = 0$. (3)

(a)(ii) Hence find x . (1)

(b) Hence find θ to one decimal place. (2)

Model solution

(a)(i) Apply the cosine rule opposite the 60° angle.

$$(3x + 10)^2 = (x + 2)^2 + (7x)^2 - 2(x + 2)(7x) \cos 60^\circ.$$

Since $\cos 60^\circ = \frac{1}{2}$,

$$9x^2 + 60x + 100 = x^2 + 4x + 4 + 49x^2 - 7x^2 - 14x = 43x^2 - 10x + 4.$$

Hence $34x^2 - 70x - 96 = 0$ and, dividing by 2,

$$17x^2 - 35x - 48 = 0$$

(a)(ii) Factorise and use the fact that $AC = 7x$ is positive.

$$(17x + 16)(x - 3) = 0 \implies x = -\frac{16}{17} \text{ or } x = 3.$$

The negative value would give a negative length, so

$$x = 3$$

(b) The side lengths are $AB = 5$, $BC = 19$ and $AC = 21$ centimetres. By the sine rule,

$$\frac{\sin \theta}{5} = \frac{\sin 60^\circ}{19}, \quad \theta = \sin^{-1} \left(\frac{5\sqrt{3}}{38} \right) = 13.1735 \dots^\circ.$$

The supplementary angle is impossible because, together with 60° , it would exceed 180° .

$$\theta = 13.2^\circ$$

Question 5

Question 5

The mass A kg of algae in a small pond is modelled by $A = pq^t$, where t is the number of weeks after the mass was first recorded. The constants p, q satisfy

$$\log_{10} A = 0.03t + 0.5.$$

- (a) Find the complete equation $A = pq^t$, with p, q each to 4 significant figures. (4)
- (b) Interpret (i) p and (ii) q with reference to the model. (2)
- (c) Find according to the model (i) the mass when $t = 8$, to the nearest 0.5 kg, and (ii) the number of weeks for the mass to reach 4 kg. (3)
- (d) State one reason why this model may be unrealistic in the long term. (1)

Model solution

(a) Take logarithms of $A = pq^t$ and compare the constant term and coefficient of t .

$$\log_{10} A = \log_{10} p + t \log_{10} q, \quad \log_{10} p = 0.5, \quad \log_{10} q = 0.03.$$

Thus $p = 10^{0.5} = 3.162277\dots$ and $q = 10^{0.03} = 1.071519\dots$

$$A = 3.162 \times 1.072^t$$

(b)(i) At $t = 0$, $A = p$.

p is the initial mass of algae in the pond, approximately 3.162 kg.

(b)(ii) The ratio $A(t+1)/A(t)$ equals q .

The mass of algae is multiplied by approximately 1.072 each week, an increase of approximately 7.2% per week.

(c)(i) Use the original logarithmic equation to retain accuracy.

$$A = 10^{0.03(8)+0.5} = 10^{0.74} = 5.4954\dots \text{ kg.}$$

5.5 kg, to the nearest 0.5 kg.

(c)(ii) Put $A = 4$ and solve for t .

$$\log_{10} 4 = 0.03t + 0.5, \quad t = \frac{\log_{10} 4 - 0.5}{0.03} = 3.40199\dots$$

3.40 weeks

(d) The model has $q > 1$, so it predicts unlimited growth.

A small pond has limited space and nutrients, so the mass of algae cannot keep increasing at this percentage rate indefinitely.

Question 6

Question 6

- (a) Find the first four terms, in ascending powers of x , of $\left(3 - \frac{2x}{9}\right)^8$, simplifying each term. (4)

$$f(x) = \left(\frac{x-1}{2x}\right) \left(3 - \frac{2x}{9}\right)^8.$$

- (b) Find the coefficient of x^2 in the series expansion of $f(x)$ as a simplified fraction. (2)

Model solution

(a) The first four binomial coefficients are 1, 8, 28, 56.

$$\left(3 - \frac{2x}{9}\right)^8 = 3^8 + 8(3^7) \left(-\frac{2x}{9}\right) + 28(3^6) \left(-\frac{2x}{9}\right)^2 + 56(3^5) \left(-\frac{2x}{9}\right)^3 + \dots$$

$$6561 - 3888x + 1008x^2 - \frac{448}{3}x^3 + \dots$$

(b) Write $(x-1)/(2x) = \frac{1}{2} - \frac{1}{2x}$. The x^2 term comes from one half of the x^2 term and $-1/(2x)$ times the x^3 term.

$$\text{Coefficient of } x^2 = \frac{1}{2}(1008) - \frac{1}{2} \left(-\frac{448}{3}\right) = 504 + \frac{224}{3}.$$

$$\frac{1736}{3}$$

Question 7

Question 7

- (a) Factorise completely $9x - x^3$. (2)
 The curve C has equation $y = 9x - x^3$.
 (b) Sketch C, showing the coordinates of its intersections with the x-axis. (2)
 The line l is $y = k$, where k is constant. Given that C and l intersect at three distinct points,
 (c) find the range of k in set notation. Solutions relying on calculator technology are not acceptable. (3)

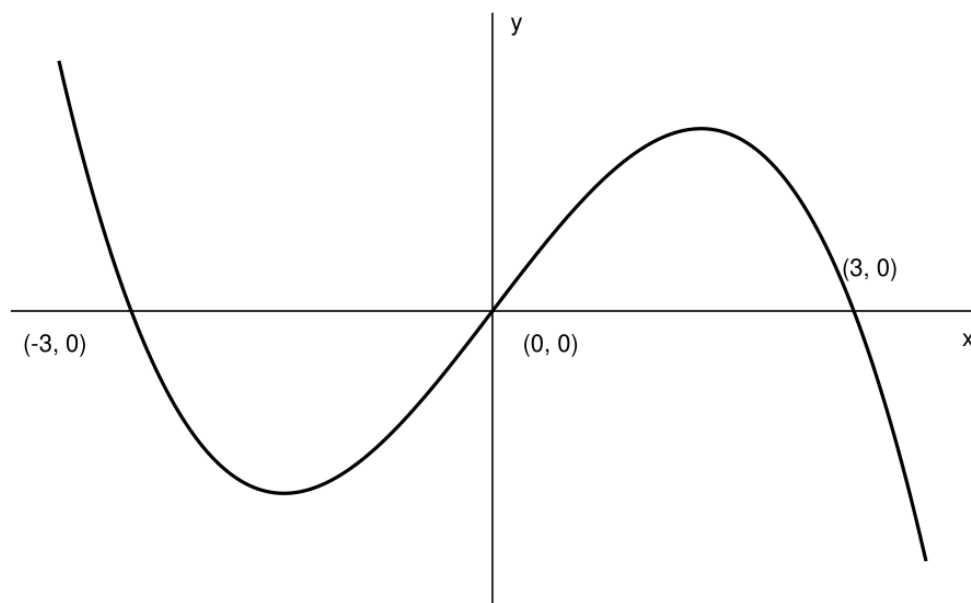
Model solution

(a) Take out x and use the difference of two squares.

$$9x - x^3 = x(9 - x^2).$$

$$x(3 - x)(3 + x)$$

(b) The roots are $-3, 0, 3$. The negative leading coefficient gives a curve descending from the upper left to the lower right, with a minimum to the left of the origin and a maximum



to the right.

The x-axis intersections are $(-3, 0)$, $(0, 0)$ and $(3, 0)$.

(c) Find the turning-point heights using calculus.

$$\frac{dy}{dx} = 9 - 3x^2 = 0 \implies x = \pm\sqrt{3}.$$

$$y(-\sqrt{3}) = -9\sqrt{3} + 3\sqrt{3} = -6\sqrt{3}, \quad y(\sqrt{3}) = 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3}.$$

The derivative is negative for $|x| > \sqrt{3}$ and positive for $|x| < \sqrt{3}$, confirming the minimum and maximum. A horizontal line meets all three branches at distinct points only when its

height is strictly between these values. Equality gives a tangency and only two distinct intersections.

$$\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\}$$

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Question 8

Question 8

Show all stages of working; solutions relying entirely on calculator technology are not acceptable. The pressure P kg/cm² inside a car tyre, t minutes after a puncture develops, is

$$P = k + 1.4e^{-0.5t}, \quad t \in \mathbb{R}, \quad t \geq 0.$$

The initial pressure is 2.2 kg/cm².

- (a) State k . (1)
- (b) Find the time for the pressure to fall to 1 kg/cm², in minutes to one decimal place. (3)
- (c) Find the rate at which the pressure is decreasing exactly 2 minutes after the puncture develops, in kg/cm² per minute to 3 significant figures. (2)

Model solution

(a) At $t = 0$, $e^0 = 1$, so $2.2 = k + 1.4$.

$$k = 0.8$$

(b) Set $P = 1$, isolate the exponential and take natural logarithms.

$$1 = 0.8 + 1.4e^{-0.5t} \implies e^{-0.5t} = \frac{0.2}{1.4} = \frac{1}{7}.$$

$$-0.5t = \ln\left(\frac{1}{7}\right) = -\ln 7, \quad t = 2 \ln 7 = 3.89182 \dots$$

$$3.9 \text{ minutes}$$

(c) Differentiate the pressure function and substitute $t = 2$.

$$\frac{dP}{dt} = 1.4(-0.5)e^{-0.5t} = -0.7e^{-0.5t}, \quad \left. \frac{dP}{dt} \right|_{t=2} = -0.7e^{-1} = -0.257515 \dots$$

The negative derivative indicates a decrease, so the rate of decrease is its positive magnitude.

$$0.258 \text{ kg/cm}^2 \text{ per minute}$$

Question 9

Question 9

- (a) Given $p = \log_3 x$, where $x > 0$, express in simplest form in terms of p : (i) $\log_3(x/9)$; (ii) $\log_3(\sqrt{x})$. (2)
 (b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3(\sqrt{x}) = -11,$$

giving a simplified fraction. Solutions relying on calculator technology are not acceptable. (4)

Model solution

(a)(i) Use the quotient rule for logarithms and $9 = 3^2$.

$$\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9 = p - 2.$$

$$p - 2$$

(a)(ii) Use $\sqrt{x} = x^{1/2}$ and the power rule.

$$\log_3(\sqrt{x}) = \frac{1}{2}\log_3 x.$$

$$\frac{p}{2}$$

(b) Substitute the expressions from part (a).

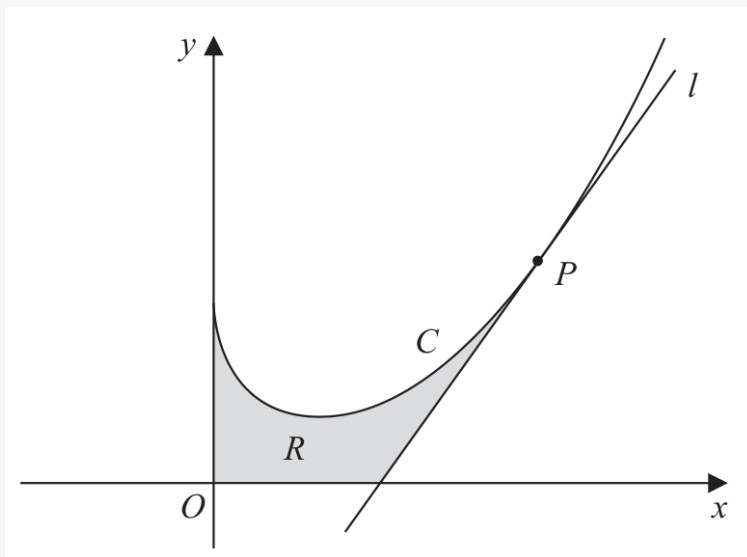
$$2(p - 2) + 3\left(\frac{p}{2}\right) = -11 \implies \frac{7}{2}p - 4 = -11 \implies p = -2.$$

Therefore $\log_3 x = -2$, so $x = 3^{-2}$, which satisfies $x > 0$.

$$x = \frac{1}{9}$$

Question 10

Question 10



Show all stages of working; solutions relying on calculator technology are not acceptable.

The curve C has equation $y = \frac{1}{3}x^2 - 2\sqrt{x} + 3$, $x \geq 0$. The point P on C has x -coordinate 4, and l is the tangent at P .

(a) Show that l has equation $13x - 6y - 26 = 0$. (5)

The shaded region R is bounded by the y -axis, C , l and the x -axis.

(b) Find the exact area of R . (5)

Model solution

(a) Differentiate, writing the square root as $x^{1/2}$.

$$\frac{dy}{dx} = \frac{2}{3}x - x^{-1/2}.$$

At $x = 4$,

$$y = \frac{16}{3} - 4 + 3 = \frac{13}{3}, \quad m = \frac{8}{3} - \frac{1}{2} = \frac{13}{6}.$$

The tangent through $(4, 13/3)$ is

$$y - \frac{13}{3} = \frac{13}{6}(x - 4) \implies 6y - 26 = 13x - 52.$$

$$13x - 6y - 26 = 0$$

(b) The tangent meets the x -axis where $13x - 26 = 0$, so $x = 2$. Subtract the triangular area beneath the tangent between $x = 2$ and $x = 4$ from the area beneath the curve between $x = 0$ and $x = 4$.

$$\int \left(\frac{1}{3}x^2 - 2x^{1/2} + 3 \right) dx = \frac{x^3}{9} - \frac{4}{3}x^{3/2} + 3x + C.$$

$$\text{Area beneath } C = \left[\frac{x^3}{9} - \frac{4}{3}x^{3/2} + 3x \right]_0^4 = \frac{64}{9} - \frac{32}{3} + 12 = \frac{76}{9}.$$

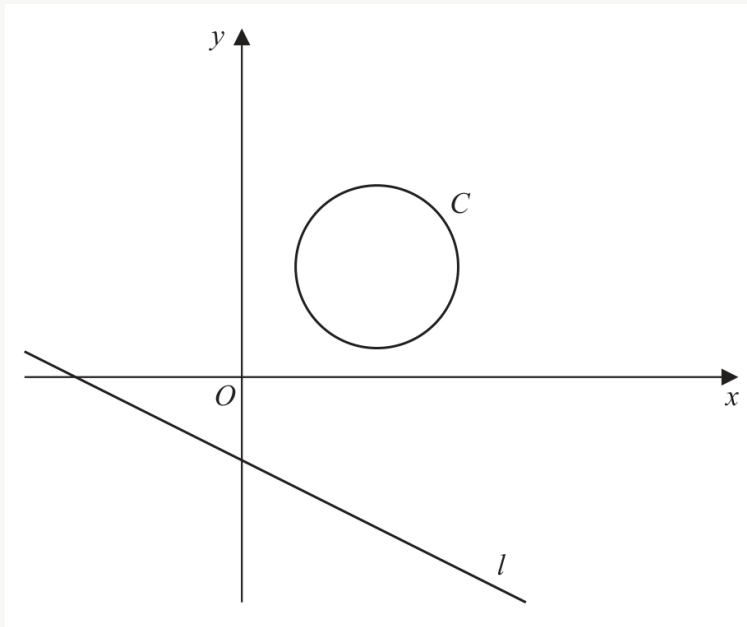
$$\text{Triangle area} = \frac{1}{2}(4-2)\frac{13}{3} = \frac{13}{3}, \quad \text{Area of R} = \frac{76}{9} - \frac{13}{3}.$$

$$\frac{37}{9} \text{ square units}$$

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Question 11

Question 11



The circle C has equation $x^2 + y^2 - 10x - 8y + 32 = 0$ and the line l has equation $2y + x + 6 = 0$.

- (a) Find (i) the centre of C and (ii) its radius. (3)
 (b) Find the shortest distance between C and l . (5)

Model solution

(a) Complete the square in both variables.

$$(x - 5)^2 - 25 + (y - 4)^2 - 16 + 32 = 0 \implies (x - 5)^2 + (y - 4)^2 = 9.$$

(i) Centre $(5, 4)$; (ii) radius 3.

(b) The shortest distance follows the perpendicular from the centre to the line, then subtracts the radius. Since l has gradient $-\frac{1}{2}$, this perpendicular has gradient 2. Its equation through the centre is

$$y - 4 = 2(x - 5), \quad y = 2x - 6.$$

Let it meet l at H . Solving simultaneously,

$$2(2x - 6) + x + 6 = 0 \implies 5x = 6, \quad x = \frac{6}{5}, \quad y = -\frac{18}{5}.$$

The distance from the centre to H is

$$\sqrt{\left(5 - \frac{6}{5}\right)^2 + \left(4 + \frac{18}{5}\right)^2} = \sqrt{\left(\frac{19}{5}\right)^2 + \left(\frac{38}{5}\right)^2} = \frac{19\sqrt{5}}{5}.$$

This exceeds the radius, so the line is outside the circle and the required gap is

$$\frac{19\sqrt{5}}{5} - 3 \text{ units} \approx 5.50 \text{ units}.$$

Question 12

Question 12

A company's metal drinks containers are modelled as closed cylinders of radius r cm, height h cm and capacity 355 cm^3 . Metal for the circular base and curved side costs 0.04 pence/cm^2 ; metal for the circular top costs 0.09 pence/cm^2 . Both metals have negligible thickness.

- Show that the total metal cost C pence per container is $C = 0.13\pi r^2 + 28.4/r$. (4)
- Use calculus to find r for which C is a minimum, to 3 significant figures. (4)
- Using d^2C/dr^2 , prove that the cost is minimised at this value. (2)
- Hence find the minimum C to the nearest integer. (2)

Model solution

(a) The negligible thickness allows the capacity and surface areas to use the same radius and height. From the cylinder volume,

$$\pi r^2 h = 355, \quad h = \frac{355}{\pi r^2}.$$

The base and top each have area πr^2 and the curved side has area $2\pi r h$. Keeping their costs separate initially,

$$C = 0.04\pi r^2 + 0.09\pi r^2 + 0.04(2\pi r h) = 0.13\pi r^2 + 0.08\pi r \left(\frac{355}{\pi r^2} \right).$$

$$C = 0.13\pi r^2 + \frac{28.4}{r}$$

(b) Differentiate with respect to the positive radius r .

$$\frac{dC}{dr} = 0.26\pi r - 28.4r^{-2}.$$

At a stationary point,

$$0.26\pi r - \frac{28.4}{r^2} = 0 \implies 0.26\pi r^3 = 28.4 \implies r = \left(\frac{1420}{13\pi} \right)^{1/3} = 3.2638\dots$$

$$r = 3.26 \text{ cm}$$

(c) Differentiate again.

$$\frac{d^2C}{dr^2} = 0.26\pi + \frac{56.8}{r^3}.$$

Both terms are positive for every $r > 0$, so the second derivative is positive at the stationary radius. The cost is strictly convex throughout the physical domain.

The stationary value found in (b) is the minimum cost.

(d) Substitute the unrounded radius from (b) into the cost model.

$$C_{\min} = 0.13\pi \left(\frac{1420}{13\pi} \right)^{2/3} + \frac{28.4}{(1420/(13\pi))^{1/3}} = 13.0520 \dots \text{ pence.}$$

$$C_{\min} = 13 \text{ pence, to the nearest integer.}$$

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Question 13

Question 13

Show all stages of working; solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos \theta} + \tan \theta \equiv \frac{\cos \theta}{1 - \sin \theta}, \quad \theta \neq (2n + 1)90^\circ, \quad n \in \mathbb{Z}. \quad (3)$$

(b) Given $\cos 2x \neq 0$, solve for $0 < x < 90^\circ$,

$$\frac{1}{\cos 2x} + \tan 2x = 3 \cos 2x,$$

giving answers to one decimal place. (5)

Model solution

(a) Use $\tan \theta = \sin \theta / \cos \theta$ and multiply by $1 - \sin \theta$ in both numerator and denominator. The stated domain makes the denominators nonzero.

$$\frac{1}{\cos \theta} + \tan \theta = \frac{1 + \sin \theta}{\cos \theta} = \frac{(1 + \sin \theta)(1 - \sin \theta)}{\cos \theta(1 - \sin \theta)} = \frac{1 - \sin^2 \theta}{\cos \theta(1 - \sin \theta)}.$$

Using $1 - \sin^2 \theta = \cos^2 \theta$ and cancelling $\cos \theta$ gives

$$\frac{1}{\cos \theta} + \tan \theta \equiv \frac{\cos \theta}{1 - \sin \theta}$$

(b) Apply the identity with $\theta = 2x$.

$$\frac{\cos 2x}{1 - \sin 2x} = 3 \cos 2x.$$

Since $\cos 2x \neq 0$, divide by $\cos 2x$ and multiply by $1 - \sin 2x$:

$$1 = 3(1 - \sin 2x), \quad \sin 2x = \frac{2}{3}.$$

The range is $0 < 2x < 180^\circ$, so there are two possibilities:

$$2x = \sin^{-1}\left(\frac{2}{3}\right) = 41.8103\dots^\circ \quad \text{or} \quad 2x = 180^\circ - 41.8103\dots^\circ = 138.1897\dots^\circ.$$

Dividing by 2 gives $20.9052\dots^\circ$ and $69.0948\dots^\circ$. Both satisfy $\cos 2x \neq 0$.

$$x = 20.9^\circ \quad \text{or} \quad x = 69.1^\circ$$

Question 14

Question 14

- (i) A student states: "If x^2 is greater than 9 then x must be greater than 3." Determine whether this is true, giving a reason. (1)
- (ii) Prove that $n^3 + 3n^2 + 2n$ is divisible by 6 for all positive integers n . (3)

Model solution

(i) A counterexample is enough to disprove the statement. For $x = -4$, $x^2 = (-4)^2 = 16 > 9$, but $x = -4$ is not greater than 3.

The statement is false.

(ii) Factorise the expression.

$$n^3 + 3n^2 + 2n = n(n^2 + 3n + 2) = n(n + 1)(n + 2).$$

These are three consecutive integers. At least one is even, so their product is divisible by 2. One of any three consecutive integers is divisible by 3, so their product is also divisible by 3. Since 2 and 3 are coprime, the product is divisible by $2 \times 3 = 6$.

$n^3 + 3n^2 + 2n$ is divisible by 6 for every positive integer n .