



Edexcel AS Mathematics

8MA0/01 - Pure Mathematics

Thursday 18 May 2023

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A curve has equation $y = \frac{2}{3}x^3 - \frac{7}{2}x^2 - 4x + 5$.

- (a) Find $\frac{dy}{dx}$, writing your answer in simplest form. (2)
- (b) Hence find the range of values of x for which y is decreasing. (4)

Model solution

(a) $\frac{dy}{dx} = 2x^2 - 7x - 4$.

$$\frac{dy}{dx} = 2x^2 - 7x - 4$$

(b) A function is decreasing where its derivative is negative. The critical values satisfy

$$2x^2 - 7x - 4 = 0 \Rightarrow (2x + 1)(x - 4) = 0,$$

so $x = -\frac{1}{2}$ or $x = 4$. Since the quadratic $2x^2 - 7x - 4$ opens upwards, it is negative between its roots.

$$-\frac{1}{2} < x < 4$$

The additive constant 5 differentiates to zero.

Question 2

Question 2

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Using the substitution $u = \sqrt{x}$, or otherwise, solve $6x + 7\sqrt{x} - 20 = 0$. (4)

Model solution

Let $u = \sqrt{x}$, so $u \geq 0$ and $x = u^2$. Then

$$6u^2 + 7u - 20 = 0$$

$$(3u - 4)(2u + 5) = 0.$$

Thus $u = \frac{4}{3}$ or $u = -\frac{5}{2}$. Since $u = \sqrt{x} \geq 0$, only $u = \frac{4}{3}$ is valid. Hence

$$x = u^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}.$$

$$x = \frac{16}{9}$$

The negative value of u cannot equal $\sqrt{x}$.

Question 3

Question 3

Three identical phone masts A, B and C have bases in the same horizontal plane. From mast C, mast A is 8.2 km away on a bearing of 072° , and mast B is 15.6 km away on a bearing of 039° .

(a) Find the distance between masts A and B, giving your answer in km to one decimal place. (3)

An engineer needs to travel from mast A to mast B.

(b) Give a reason why the answer to part (a) is unlikely to be an accurate value for the distance the engineer travels. (1)

Model solution

(a) The included angle is

$$\angle ACB = 72^\circ - 39^\circ = 33^\circ.$$

By the cosine rule,

$$AB^2 = 8.2^2 + 15.6^2 - 2(8.2)(15.6) \cos 33^\circ,$$

so $AB = 9.790 \dots \text{ km}$.

$$AB = 9.8 \text{ km (to 1 d.p.)}$$

(b) This is the straight-line distance. Roads or obstacles are unlikely to allow the engineer to travel in a straight line, so the distance travelled will generally be greater.

The route is unlikely to be straight.

The bearing diagram gives the included angle at C; the cosine rule then applies to triangle ACB.

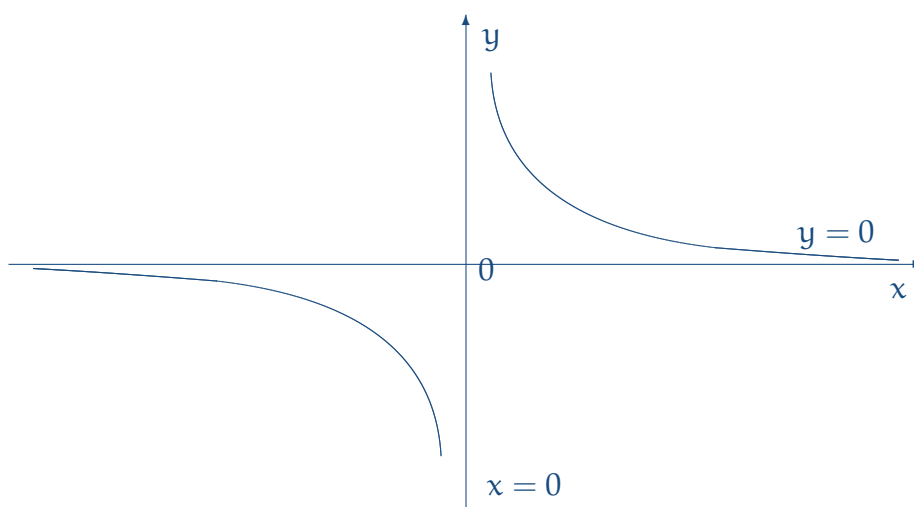
Question 4

Question 4

- (a) Sketch the curve $y = \frac{k}{x}$, $x \neq 0$, where k is a positive constant. (2)
- (b) Hence or otherwise, solve $\frac{16}{x} \leq 2$. (3)

Model solution

(a) Since $k > 0$, y has the same sign as x . The sketch is a rectangular hyperbola with branches in quadrants I and III, with the coordinate axes as asymptotes.



Branches in quadrants I and III; asymptotes $x = 0$ and $y = 0$.

(b) If $x < 0$, then $16/x < 0$, so the inequality is true. If $x > 0$, multiplying by x gives

$$16 \leq 2x \Rightarrow x \geq 8.$$

$$x < 0 \text{ or } x \geq 8$$

The sign of x must be considered before multiplying an inequality by x .

Question 5

Question 5

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The finite region R is bounded by $y = 4x^2 + 3$, the y -axis and the line $y = 23$. Show that the exact area of R is $k\sqrt{5}$, where k is a rational constant to be found. (5)

Model solution

The curve meets $y = 23$ when

$$4x^2 + 3 = 23 \Rightarrow x^2 = 5.$$

The relevant positive limit is $x = \sqrt{5}$. Therefore

$$\begin{aligned} \text{Area}(R) &= \int_0^{\sqrt{5}} (23 - (4x^2 + 3)) \, dx \\ &= \left[20x - \frac{4}{3}x^3 \right]_0^{\sqrt{5}} \\ &= 20\sqrt{5} - \frac{4}{3}(\sqrt{5})^3 \\ &= 20\sqrt{5} - \frac{20}{3}\sqrt{5} = \frac{40}{3}\sqrt{5}. \end{aligned}$$

$$\text{Area}(R) = \frac{40}{3}\sqrt{5}, \text{ so } k = \frac{40}{3}$$

On $0 \leq x \leq \sqrt{5}$, the horizontal line is above the curve.

Question 6

Question 6

The circle C has equation $x^2 + y^2 - 6x + 10y + k = 0$, where k is a constant.

(a) Find the coordinates of the centre of C. (2)

Given that C does not cut or touch the x -axis,

(b) find the range of possible values for k . (3)

Model solution

(a) Completing the square gives

$$(x - 3)^2 + (y + 5)^2 = 34 - k.$$

Hence the centre is $(3, -5)$.

$$(3, -5)$$

(b) The centre is 5 units below the x -axis. For the circle not to touch or cut the x -axis, its radius must be strictly less than 5, but positive:

$$0 < 34 - k < 25.$$

Solving gives $9 < k < 34$.

$$9 < k < 34$$

The upper bound excludes a point circle, while the lower bound excludes tangency to the x -axis.

Question 7

Question 7

For a particular car, 40 litres remain after 80 km and 25 litres remain after 200 km. Using a linear model, V is the volume of fuel remaining in litres and d is the distance travelled in km.

(a) Find an equation linking V with d . (4)

Given that a journey starts with a full tank and continues until the car runs out of fuel, find, according to the model,

(b)(i) the initial volume of fuel; (ii) the distance travelled. (3)

In fact the car travelled 320 km. (c) Evaluate the model in light of this information. (1)

Model solution

(a) Let $V = ad + b$. Using $(d, V) = (80, 40)$ and $(200, 25)$,

$$40 = 80a + b, \quad 25 = 200a + b.$$

Subtracting gives $-15 = 120a$, hence $a = -\frac{1}{8}$. Then $40 = -10 + b$, so $b = 50$.

$$V = -\frac{1}{8}d + 50$$

(b) At $d = 0$, $V = 50$, so the initial volume is 50 litres. When the tank is empty, $V = 0$:

$$0 = -\frac{1}{8}d + 50 \Rightarrow d = 400.$$

(i) 50 litres (ii) 400 km

(c) The observed distance, 320 km, is substantially less than the model prediction of 400 km. Therefore this is a poor model for this journey.

The model is poor because it overestimates the distance by 80 km.

A linear model assumes the fuel remaining falls at a constant rate.

Question 8

Question 8

A curve C has equation $y = f(x)$, where $f(x)$ is quadratic, and cuts the x -axis at 0 and 6. A line l cuts the y -axis at 60 and intersects C at $(10, 80)$. Use inequalities to define the shaded region R between C and l . (5)

Model solution

For l , the gradient is

$$\frac{80 - 60}{10 - 0} = 2,$$

so l has equation $y = 2x + 60$.

Since C has roots 0 and 6, write $y = ax(x - 6)$. Using $(10, 80)$,

$$80 = a(10)(4),$$

so $a = 2$, and C has equation $y = 2x(x - 6)$.

The shaded region is above the curve and below the line:

$$2x(x - 6) \leq y \leq 2x + 60$$

Both boundaries are included, as they form the edge of the shaded region.

Question 9

Question 9

Using the laws of logarithms, solve $2\log_5(3x - 2) - \log_5 x = 2$.

(5)

Model solution

The logarithms require $x > 0$ and $3x - 2 > 0$, so $x > \frac{2}{3}$. Applying log laws,

$$\log_5 \left(\frac{(3x - 2)^2}{x} \right) = \log_5 25.$$

Thus

$$(3x - 2)^2 = 25x,$$

$$9x^2 - 12x + 4 = 25x,$$

$$9x^2 - 37x + 4 = 0 = (9x - 1)(x - 4).$$

This gives $x = \frac{1}{9}$ or $x = 4$. The value $\frac{1}{9}$ is outside the domain $x > \frac{2}{3}$.

$$x = 4$$

Checking the domain is essential because logarithms are defined only for positive arguments.

Question 10

Question 10

The line l_1 has equation $y = \frac{3}{5}x + 6$. The line l_2 is perpendicular to l_1 and passes through $B(8,0)$.

(a) Show that an equation for l_2 is $\frac{5}{3}x + y = \frac{40}{3}$. (3)

The lines intersect at C , and l_1 crosses the x -axis at A . (b) Find the exact area of triangle ABC , as a fully simplified fraction. (5)

Model solution

(a) The gradient of l_1 is $\frac{3}{5}$, so the perpendicular gradient is $-\frac{5}{3}$. Through $(8,0)$,

$$y - 0 = -\frac{5}{3}(x - 8) = -\frac{5}{3}x + \frac{40}{3}.$$

Therefore $\frac{5}{3}x + y = \frac{40}{3}$, as required.

(b) At A , $y = 0$, so

$$0 = \frac{3}{5}x + 6 \Rightarrow A = (-10, 0).$$

At C , substitute $y = \frac{3}{5}x + 6$ into $\frac{5}{3}x + y = \frac{40}{3}$:

$$\frac{5}{3}x + \frac{3}{5}x + 6 = \frac{40}{3}.$$

Hence $x = \frac{55}{17}$ and $y = \frac{135}{17}$. The base AB has length 18, and the perpendicular height is $\frac{135}{17}$. Thus

$$\text{Area} = \frac{1}{2} \times 18 \times \frac{135}{17} = \frac{1215}{17}.$$

$$\frac{1215}{17}$$

Using AB , which lies on the x -axis, makes the y -coordinate of C the perpendicular height.

Question 11

Question 11

The height h metres of a plant, t years after it was first measured, is modelled by $h = 2.3 - 1.7e^{-0.2t}$, where $t \geq 0$.

- (a) Find the height when it was first measured. (2)
- (b) Show that exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year. (3)
- (c) Deduce the limiting height according to the model. (1)

Model solution

(a) At $t = 0$,

$$h = 2.3 - 1.7e^0 = 0.6.$$

0.6 m

(b) $\frac{dh}{dt} = (-1.7)(-0.2)e^{-0.2t} = 0.34e^{-0.2t}$. Hence at $t = 4$,

$$\frac{dh}{dt} = 0.34e^{-0.8} = 0.15277 \dots \text{ m year}^{-1} = 15.277 \dots \text{ cm year}^{-1}.$$

$$\frac{dh}{dt} \approx 15.3 \text{ cm per year}$$

(c) As $t \rightarrow \infty$, $e^{-0.2t} \rightarrow 0$, so $h \rightarrow 2.3$.

2.3 m

The rate of growth is the derivative of height with respect to time.

Question 12

Question 12

In this question you must show detailed reasoning. Solutions relying entirely on calculator technology are not acceptable.

- (a) Show that $4 \tan x = 5 \cos x$ can be written as $5 \sin^2 x + 4 \sin x - 5 = 0$. (3)
 (b) Hence solve $4 \tan x = 5 \cos x$, for $0^\circ < x \leq 360^\circ$, giving your answers to one decimal place. (4)
 (c) Hence find the number of solutions of $4 \tan 3x = 5 \cos 3x$ in $0^\circ < x \leq 1800^\circ$, explaining briefly the reason for your answer. (2)

Model solution

(a) Using $\tan x = \frac{\sin x}{\cos x}$,

$$4 \frac{\sin x}{\cos x} = 5 \cos x \Rightarrow 4 \sin x = 5 \cos^2 x.$$

Since $\cos^2 x = 1 - \sin^2 x$,

$$4 \sin x = 5(1 - \sin^2 x) \Rightarrow 5 \sin^2 x + 4 \sin x - 5 = 0.$$

(b) Solving this quadratic,

$$\sin x = \frac{-4 \pm \sqrt{4^2 - 4(5)(-5)}}{10} = \frac{-2 \pm \sqrt{29}}{5}.$$

Only $\sin x = \frac{-2 + \sqrt{29}}{5} = 0.677 \dots$ is possible. Therefore

$$x = \sin^{-1}(0.677 \dots) = 42.6^\circ \quad \text{or} \quad x = 180^\circ - 42.6^\circ = 137.4^\circ.$$

$$x = 42.6^\circ, 137.4^\circ$$

(c) Put $u = 3x$. The interval $0^\circ < x \leq 1800^\circ$ gives $0^\circ < u \leq 5400^\circ$. Since $5400^\circ = 15 \times 360^\circ$, this contains 15 complete cycles, with two solutions per cycle from part (b). Thus the number of solutions is $15 \times 2 = 30$.

30 solutions

The substitution $u = 3x$ changes the interval before the two solutions from part (b) are counted in each complete cycle.

Question 13

Question 13

Relative to a fixed origin O , $\overrightarrow{OA} = 10\mathbf{i} - 3\mathbf{j}$, $\overrightarrow{OB} = -8\mathbf{i} + 9\mathbf{j}$, and $\overrightarrow{OC} = -2\mathbf{i} + p\mathbf{j}$, where p is a constant.

(a) Find $\overrightarrow{AB}$. (2)

(b) Find AB , giving your answer as a fully simplified surd. (2)

Given that A , B and C lie on a straight line, (c)(i) find p ; (ii) state the ratio of the area of triangle AOC to the area of triangle AOB . (3)

Model solution

(a)

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-8\mathbf{i} + 9\mathbf{j}) - (10\mathbf{i} - 3\mathbf{j}) = -18\mathbf{i} + 12\mathbf{j}.$$

$$\overrightarrow{AB} = -18\mathbf{i} + 12\mathbf{j}$$

(b)

$$AB = \sqrt{(-18)^2 + 12^2} = \sqrt{468} = 6\sqrt{13}.$$

$$6\sqrt{13}$$

(c) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 6\mathbf{i} + (p - 9)\mathbf{j}$. Since BC is parallel to AB , let $\overrightarrow{AB} = \lambda \overrightarrow{BC}$. From the $\mathbf{i}$ -components,

$$-18 = 6\lambda \Rightarrow \lambda = -3.$$

From the $\mathbf{j}$ -components,

$$12 = -3(p - 9) \Rightarrow p = 5.$$

Point C divides AB in the ratio $AC : CB = 2 : 1$, so triangles AOC and AOB have the same perpendicular height from O to line AB , and their area ratio is $AC : AB = 2 : 3$.

$$(i) p = 5 \quad (ii) 2 : 3$$

For triangles sharing the same altitude to line AB , their areas are in the ratio of their bases on that line.

Question 14

Question 14

Find, in simplest form, the coefficient of x^5 in the expansion of $(5 + 8x^2) \left(3 - \frac{1}{2}x\right)^6$. (5)

Model solution

An x^5 term arises from 5 times the x^5 term of the binomial, and from $8x^2$ times the x^3 term of the binomial.

$$[x^5] \left(3 - \frac{1}{2}x\right)^6 = \binom{6}{5} 3 \left(-\frac{1}{2}\right)^5 = -\frac{9}{16},$$

$$[x^3] \left(3 - \frac{1}{2}x\right)^6 = \binom{6}{3} 3^3 \left(-\frac{1}{2}\right)^3 = -\frac{135}{2}.$$

Therefore the required coefficient is

$$5 \left(-\frac{9}{16}\right) + 8 \left(-\frac{135}{2}\right) = -\frac{45}{16} - 540 = -\frac{8685}{16}.$$

$$-\frac{8685}{16}$$

Only the two stated combinations of powers can produce x^5 .

Question 15

Question 15

In this question you must show detailed reasoning. Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation $y = 8 - 10x + 6x^2 - x^3$. The curve C_2 has equation $y = x^2 - 12x + 14$.

(a) Verify that when $x = 1$, C_1 and C_2 intersect. (2)

The curves also intersect when $x = k$. Given that $k < 0$, (b) use algebra to find the exact value of k . (5)

Model solution

(a) When $x = 1$,

$$C_1 : y = 8 - 10 + 6 - 1 = 3, \quad C_2 : y = 1 - 12 + 14 = 3.$$

Both curves contain $(1, 3)$, so they intersect when $x = 1$.

(b) At an intersection,

$$8 - 10x + 6x^2 - x^3 = x^2 - 12x + 14.$$

Rearranging,

$$x^3 - 5x^2 - 2x + 6 = 0.$$

Since $x = 1$ is a root,

$$x^3 - 5x^2 - 2x + 6 = (x - 1)(x^2 - 4x - 6).$$

Thus $x^2 - 4x - 6 = 0$, so

$$x = \frac{4 \pm \sqrt{16 + 24}}{2} = 2 \pm \sqrt{10}.$$

Since $k < 0$, the required root is $2 - \sqrt{10}$.

$$k = 2 - \sqrt{10}$$

The positive root is rejected because the question specifies $k < 0$.

Question 16

Question 16

A curve has equation $y = f(x)$, $x \geq 0$. Given that $f'(x) = 4x + a\sqrt{x} + b$, where a and b are constants, the curve has a stationary point at $(4, 3)$, and it meets the y -axis at -5 , find $f(x)$ in simplest form. (6)

Model solution

At the stationary point, $f'(4) = 0$, so

$$16 + 2a + b = 0.$$

Integrating,

$$f(x) = 2x^2 + \frac{2a}{3}x^{3/2} + bx + c.$$

Since the curve meets the y -axis at -5 , $c = -5$. Also $f(4) = 3$, hence

$$32 + \frac{16a}{3} + 4b - 5 = 3,$$

so $\frac{16a}{3} + 4b = -24$. Together with $2a + b = -16$, this gives $a = -15$ and $b = 14$. Therefore

$$f(x) = 2x^2 - 10x^{3/2} + 14x - 5.$$

$$f(x) = 2x^2 - 10x\sqrt{x} + 14x - 5$$

Both conditions at $x = 4$ are needed: stationarity gives $f'(4) = 0$, and the point gives $f(4) = 3$.

Question 17

Question 17

In this question p and q are positive integers with $q > p$.

Statement 1: $q^3 - p^3$ is never a multiple of 5.

(a) Show, by means of a counterexample, that Statement 1 is not true. (1)

Statement 2: When p and q are consecutive even integers, $q^3 - p^3$ is a multiple of 8.

(b) Prove, using algebra, that Statement 2 is true. (4)

Model solution

(a) Take $p = 1$ and $q = 6$. Then

$$q^3 - p^3 = 6^3 - 1^3 = 215 = 5 \times 43.$$

Thus Statement 1 is not true.

For example, $p = 1$, $q = 6$.

(b) Let the consecutive even integers be $p = 2n$ and $q = 2n + 2$, where n is a positive integer. Then

$$\begin{aligned} q^3 - p^3 &= (2n + 2)^3 - (2n)^3 \\ &= (8n^3 + 24n^2 + 24n + 8) - 8n^3 \\ &= 24n^2 + 24n + 8 \\ &= 8(3n^2 + 3n + 1). \end{aligned}$$

Since $3n^2 + 3n + 1$ is an integer, $q^3 - p^3$ is a multiple of 8. Hence Statement 2 is true.

$$q^3 - p^3 = 8(3n^2 + 3n + 1), \text{ so it is a multiple of 8.}$$

The factorisation by 8 is valid for every integer n , completing the proof.