



Edexcel AS Mathematics

8MA0/01 - Pure Mathematics

Thursday 16 May 2024

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2024 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Find $\int \frac{2\sqrt{x}-3}{x^2} dx$, giving your answer in simplest form.

Model solution

Write each term using an index.

$$\int \frac{2\sqrt{x}-3}{x^2} dx = \int (2x^{-3/2} - 3x^{-2}) dx = -4x^{-1/2} + 3x^{-1} + C.$$

$$-\frac{4}{\sqrt{x}} + \frac{3}{x} + C$$

(4)

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Question 2

Question 2

$f(x) = 2x^3 - 3ax^2 + bx + 8a$, where a and b are constants. Given that $(x - 4)$ is a factor of $f(x)$:

(a) use the factor theorem to show that $10a = 32 + b$.

Given also that $(x - 2)$ is a factor of $f(x)$:

(b) express $f(x)$ as $(2x + k)(x - 4)(x - 2)$, finding k .

(c) Hence (i) state the number of real roots of $f(x) = 0$, and (ii) write down the largest root of $f(\frac{1}{3}x) = 0$.

Model solution

(a) Since $(x - 4)$ is a factor, the factor theorem gives $f(4) = 0$.

$$2(4)^3 - 3a(4)^2 + 4b + 8a = 0 \Rightarrow 128 - 48a + 4b + 8a = 0 \Rightarrow 10a = 32 + b.$$

((a) 2)

(b) Similarly, $f(2) = 0$.

$$2(2)^3 - 3a(2)^2 + 2b + 8a = 0 \Rightarrow 8 - 12a + 2b + 8a = 0 \Rightarrow 2a - b = 4.$$

Solving this with $10a - b = 32$ gives $a = 3$ and $b = -2$. Thus

$$f(x) = 2x^3 - 9x^2 - 2x + 24 = (2x + 3)(x - 4)(x - 2).$$

Hence $k = 3$.

((b) 4)

(c) The three factors give the roots $-\frac{3}{2}, 2, 4$, all real. For $f(\frac{1}{3}x) = 0$, multiply each root by 3.

(i) 3 real roots (ii) 12

((c) 2)

Question 3

Question 3

Relative to origin O, $\vec{OP} = 9\mathbf{i} - 8\mathbf{j}$ and $\vec{OQ} = 3\mathbf{i} - 5\mathbf{j}$.

(a) Find $\vec{PQ}$. Given $\vec{QR} = 9\mathbf{i} + 18\mathbf{j}$, (b) show that $\angle PQR = 90^\circ$. Given also $\vec{PS} = 3\vec{QR}$, (c) find the exact area of PQRS.

Model solution

$$\vec{PQ} = \vec{OQ} - \vec{OP} = (3 - 9)\mathbf{i} + (-5 - (-8))\mathbf{j} = -6\mathbf{i} + 3\mathbf{j}.$$

$$\vec{PQ} = -6\mathbf{i} + 3\mathbf{j}$$

((a) 2)

The gradients of PQ and QR are, respectively,

$$\frac{3}{-6} = -\frac{1}{2} \quad \text{and} \quad \frac{18}{9} = 2.$$

Their product is -1 , so the lines are perpendicular. Therefore $\angle PQR = 90^\circ$.

((b) 2)

The parallel sides of trapezium PQRS are QR and PS.

$$PQ = \sqrt{(-6)^2 + 3^2} = 3\sqrt{5}, \quad QR = \sqrt{9^2 + 18^2} = 9\sqrt{5}, \quad PS = 3QR = 27\sqrt{5}.$$

$$\text{Area} = \frac{1}{2}(PQ)(QR + PS) = \frac{1}{2}(3\sqrt{5})(36\sqrt{5}) = 270.$$

270 square units

((c) 4)

Question 4

Question 4

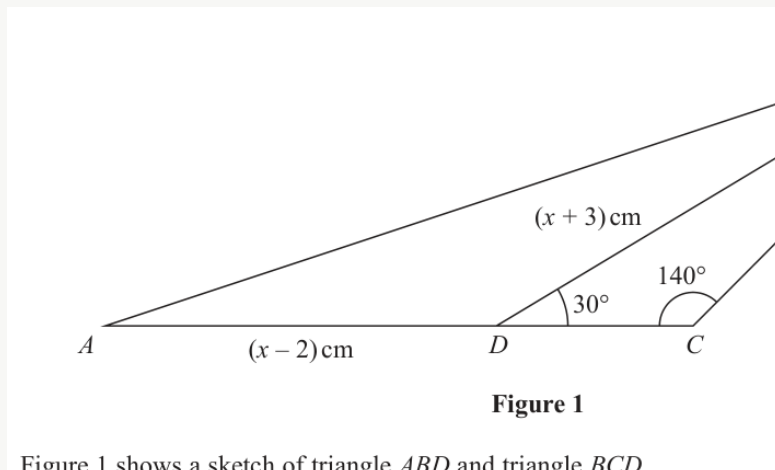


Figure 1 shows a sketch of triangle ABD and triangle BCD

In the diagram, A, D, C are collinear, $BD = (x+3)$ cm, $BC = x$ cm, $\angle BDC = 30^\circ$, and $\angle BCD = 140^\circ$.

(a) Show that $x = 10.5$, correct to 3 significant figures. Given also $AD = (x-2)$ cm, (b) find AB , to 3 significant figures.

Model solution

In triangle BCD , the sine rule gives

$$\frac{x}{\sin 30^\circ} = \frac{x+3}{\sin 140^\circ}$$

$$x \sin 140^\circ = (x+3) \sin 30^\circ \Rightarrow x = \frac{3 \sin 30^\circ}{\sin 140^\circ - \sin 30^\circ} = 10.505 \dots$$

$$x = 10.5 \text{ (3 s.f.)}$$

((a) 3)

Since ADC is a straight line, $\angle ADB = 180^\circ - 30^\circ = 150^\circ$. Also $AD = 8.5$ cm and $BD = 13.5$ cm. By the cosine rule,

$$AB^2 = 8.5^2 + 13.5^2 - 2(8.5)(13.5) \cos 150^\circ, \quad AB = 21.299 \dots$$

$$AB = 21.3 \text{ cm (3 s.f.)}$$

((b) 2)

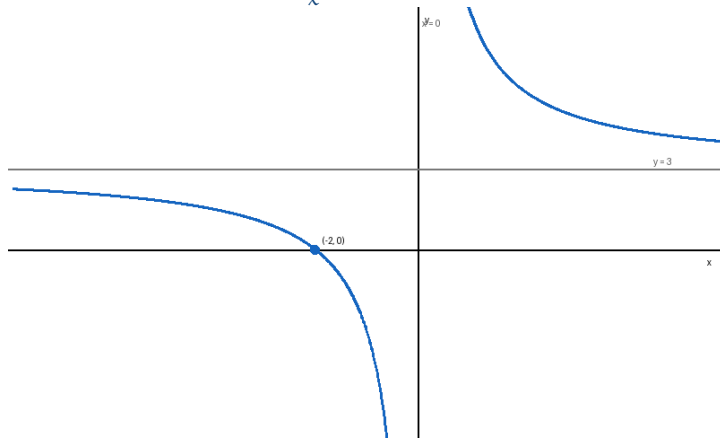
Question 5

Question 5

C_1 has equation $y = \frac{6}{x} + 3$. (a) (i) Sketch C_1 , stating any intercepts; (ii) state its asymptotes. C_2 has equation $y = 3x^2 - 4x - 10$. (b) Show that intersections satisfy $3x^3 - 4x^2 - 13x - 6 = 0$. Given one intersection has $x = -\frac{2}{3}$, (c) find algebraically the other x -coordinates.

Model solution

(a) C_1 is a reciprocal curve translated up 3 units. It meets the x -axis when $0 = \frac{6}{x} + 3$, so at $(-2, 0)$, and it does not meet the y -axis.



Asymptotes: $x = 0$ and $y = 3$.

((a) 3)

$$\frac{6}{x} + 3 = 3x^2 - 4x - 10 \Rightarrow \frac{6}{x} = 3x^2 - 4x - 13$$

$$\Rightarrow 6 = 3x^3 - 4x^2 - 13x \Rightarrow 3x^3 - 4x^2 - 13x - 6 = 0.$$

((b) 2)

(c) Since $x = -\frac{2}{3}$ is a root, $3x + 2$ is a factor.

$$3x^3 - 4x^2 - 13x - 6 = (3x + 2)(x^2 - 2x - 3) = (3x + 2)(x - 3)(x + 1).$$

The other x -coordinates are $x = -1$ and $x = 3$.

((c) 4)

Question 6

Question 6

The expansion of $(1 + ax)^{12}$, up to and including x^2 , is $1 - \frac{15}{2}x + kx^2$. (a) Show $a = -\frac{5}{8}$. (b) Find k . (c) Use the expansion to estimate $\left(\frac{17}{16}\right)^{12}$, to 4 decimal places.

Model solution

$$(1 + ax)^{12} = 1 + 12ax + \binom{12}{2}a^2x^2 + \dots$$

Comparing coefficients of x , $12a = -\frac{15}{2}$, so

$$a = -\frac{5}{8}$$

((a) 2)

$$k = \binom{12}{2} \left(-\frac{5}{8}\right)^2 = 66 \cdot \frac{25}{64} = \frac{825}{32}.$$

$$k = \frac{825}{32}$$

((b) 2)

Choose x so that $1 - \frac{5}{8}x = \frac{17}{16}$, giving $x = -\frac{1}{10}$.

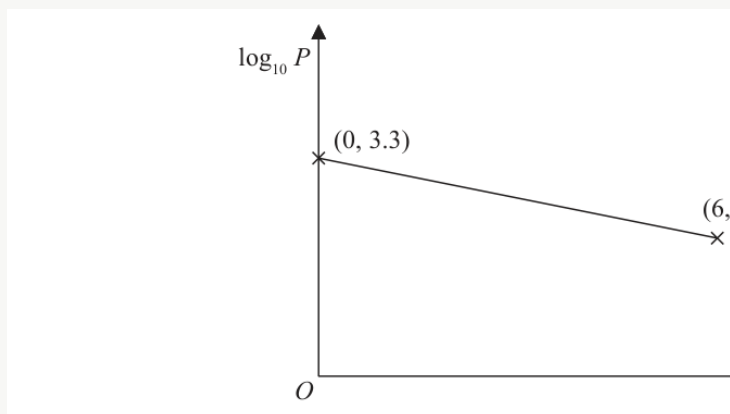
$$\left(\frac{17}{16}\right)^{12} \approx 1 - \frac{15}{2} \left(-\frac{1}{10}\right) + \frac{825}{32} \left(-\frac{1}{10}\right)^2 = 2.0078125.$$

$$2.0078$$

((c) 2)

Question 7

Question 7



Smoke concentration P ppm at distance x km is modelled by the straight-line relationship between $\log_{10} P$ and x , through $(0, 3.3)$ and $(6, 2.1)$. (a) Find $P = ab^x$, giving a, b to 4 significant figures. (b) Interpret ab .

Model solution

$$\text{gradient} = \frac{2.1 - 3.3}{6} = -0.2, \quad \log_{10} P = 3.3 - 0.2x.$$

$$P = 10^{3.3-0.2x} = 10^{3.3}(10^{-0.2})^x.$$

$$10^{3.3} = 1995.262\dots, \quad 10^{-0.2} = 0.630957\dots$$

$$P = 1995(0.6310)^x, \text{ so } a = 1995, \text{ } b = 0.6310.$$

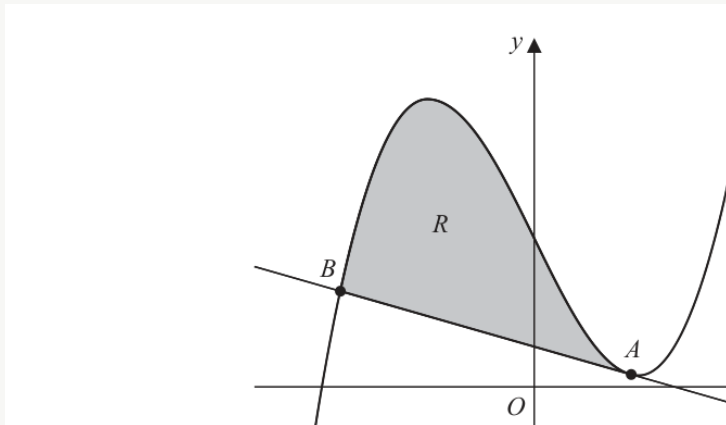
((a) 4) When $x = 1$, $P = ab$.

ab is the modelled smoke concentration, in ppm, 1 km from the chimney.

((b) 1)

Question 8

Question 8



C has equation $y = x^3 - 14x + 23$. Line l , tangent at A , is $y = -2x + 7$. (a) Show that the x -coordinate of A is 2. (b) Verify that the other intersection B has $x = -4$. (c) Show, by algebraic integration, that the bounded area is 108.

Model solution

$$\frac{dy}{dx} = 3x^2 - 14.$$

At the tangent, this equals -2 , so

$$3x^2 - 14 = -2 \Rightarrow x^2 = 4.$$

The tangent point shown has positive coordinate, hence

$$x = -2 \text{ cannot be } A, \text{ as } y_C(-2) = 43 \neq 11 = y_l(-2).$$

$$x = 2$$

((a) 3)

$$\text{At } x = -4, \quad x^3 - 14x + 23 = -64 + 56 + 23 = 15, \quad -2(-4) + 7 = 15.$$

Both equations give the same y -value, so $x = -4$ is the coordinate of B .

((b) 2)

Between -4 and 2 , the curve is above the line.

$$\begin{aligned} \text{Area} &= \int_{-4}^2 [(x^3 - 14x + 23) - (-2x + 7)] \, dx = \int_{-4}^2 (x^3 - 12x + 16) \, dx \\ &= \left[\frac{x^4}{4} - 6x^2 + 16x \right]_{-4}^2 = 12 - (-96) = 108. \end{aligned}$$

$$\text{Area} = 108 \text{ square units}$$

((c) 5)

Question 9

Question 9

$p = \log_a 16$ and $q = \log_a 25$. Find, in terms of p and/or q : (a) $\log_a 256$, (b) $\log_a 100$, (c) $\log_a 80 \times \log_a 3.2$.

Model solution

$$\log_a 256 = \log_a (16^2) = 2 \log_a 16.$$

$$(a) \ 2p$$

((a) 1)

$$\log_a 100 = \log_a (4 \times 25) = \log_a 4 + \log_a 25 = \frac{1}{2} \log_a 16 + \log_a 25.$$

$$(b) \ \frac{1}{2}p + q$$

((b) 2)

$$\log_a 80 = \log_a (16 \times 5) = p + \frac{1}{2}q, \quad \log_a 3.2 = \log_a \left(\frac{16}{5}\right) = p - \frac{1}{2}q.$$

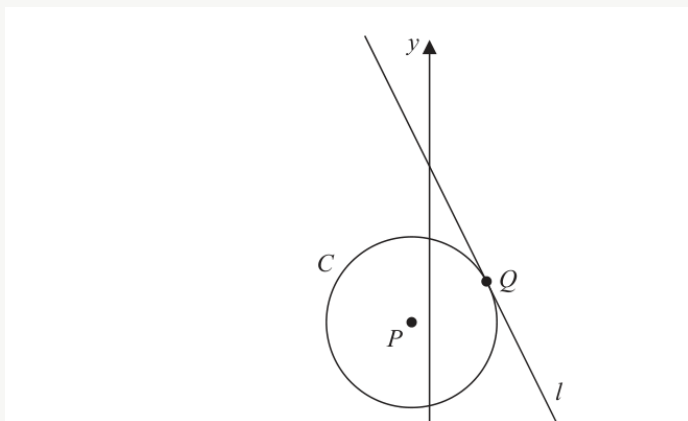
$$\log_a 80 \times \log_a 3.2 = \left(p + \frac{1}{2}q\right) \left(p - \frac{1}{2}q\right) = p^2 - \frac{1}{4}q^2.$$

$$(c) \ p^2 - \frac{1}{4}q^2$$

((c) 2)

Question 10

Question 10



Circle C has centre $P(-1, k+8)$; $Q(3, k^2-2k)$ lies on C; $k > 0$; and the tangent at Q has gradient -2 . (a) Show that $k^2 - 3k - 10 = 0$. (b) Hence find an equation for C.

Model solution

(a) The radius PQ is perpendicular to the tangent, so its gradient is the negative reciprocal of -2 , namely $\frac{1}{2}$.

$$\frac{(k^2 - 2k) - (k + 8)}{3 - (-1)} = \frac{1}{2} \Rightarrow \frac{k^2 - 3k - 8}{4} = \frac{1}{2}$$

$$\Rightarrow k^2 - 3k - 8 = 2 \Rightarrow k^2 - 3k - 10 = 0.$$

((a) 4)

$$(k - 5)(k + 2) = 0.$$

As $k > 0$, $k = 5$. Thus $P = (-1, 13)$ and $Q = (3, 15)$. The radius is

$$PQ = \sqrt{(3 + 1)^2 + (15 - 13)^2} = \sqrt{20}.$$

$$(x + 1)^2 + (y - 13)^2 = 20$$

((b) 4)

Question 11

Question 11

The price per gram of metal A is $V_A = 100 + 20e^{0.04t}$; that of metal B is $V_B = pe^{-0.02t}$, where t is months after monitoring began and $p > 0$. Given $V_B = 2V_A$ when $t = 0$: (a) find p . When $t = T$, the rate of increase of A equals the rate of decrease of B. (b) find T , to 1 decimal place.

Model solution

$$V_A(0) = 100 + 20 = 120, \quad V_B(0) = p.$$

Since $V_B(0) = 2V_A(0)$,

$$p = 240$$

((a) 2)

$$\frac{dV_A}{dt} = 0.8e^{0.04t}, \quad \frac{dV_B}{dt} = -0.02(240)e^{-0.02t} = -4.8e^{-0.02t}.$$

Equating the positive rate of increase of A to the magnitude of the rate of decrease of B,

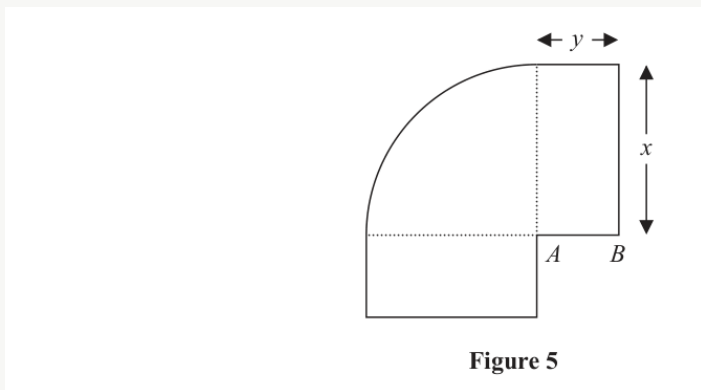
$$0.8e^{0.04T} = 4.8e^{-0.02T} \Rightarrow e^{0.06T} = 6 \Rightarrow T = \frac{\ln 6}{0.06} = 29.862 \dots$$

$$T = 29.9 \text{ months}$$

((b) 4)

Question 12

Question 12



A pool consists of a quarter-circle of radius x metres joined to two equal rectangles, each x m by y m. Its surface area is 100 m^2 . (a) Show that $P = 2x + \frac{200}{x}$. (b) Find the stationary value of x . (c) Prove this gives a minimum perimeter. Access side AB must be at least 1 m: (d) decide whether the minimum-perimeter pool is suitable.

Model solution

(a) The total area is the two rectangles plus the quarter-circle.

$$2xy + \frac{1}{4}\pi x^2 = 100 \Rightarrow y = \frac{400 - \pi x^2}{8x}.$$

The perimeter is two straight x -lengths, the quarter-circle arc, and four y -lengths:

$$P = 2x + \frac{1}{2}\pi x + 4y = 2x + \frac{1}{2}\pi x + 4\left(\frac{400 - \pi x^2}{8x}\right) = 2x + \frac{200}{x}.$$

$$P = 2x + \frac{200}{x}$$

((a) 5)

$$\frac{dP}{dx} = 2 - \frac{200}{x^2}.$$

$$2 - \frac{200}{x^2} = 0 \Rightarrow 2x^2 = 200 \Rightarrow x^2 = 100.$$

As a length is positive,

$$x = 10 \text{ m}$$

((b) 4)

$$\frac{d^2P}{dx^2} = \frac{400}{x^3}.$$

At $x = 10$, $\frac{d^2P}{dx^2} = 0.4 > 0$. Hence the stationary perimeter is a minimum.

((c) 2)

$$y = \frac{400 - \pi(10)^2}{8(10)} = \frac{400 - 100\pi}{80} = 1.073\dots m.$$

Since $AB = y > 1\text{ m}$,

the minimum-perimeter pool would be suitable.

((d) 2)

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Question 13

Question 13

(a) Show that $\sin \theta(7 \sin \theta - 4 \cos \theta) = 4$ can be written as $3 \tan^2 \theta - 4 \tan \theta - 4 = 0$. (b) Hence solve $\sin x(7 \sin x - 4 \cos x) = 4$ for $0 < x < 360^\circ$, to 1 decimal place. (c) Hence find the smallest solution of $\sin 4\alpha(7 \sin 4\alpha - 4 \cos 4\alpha) = 4$ for $720^\circ < \alpha < 1080^\circ$, to 1 decimal place.

Model solution

$$7 \sin^2 \theta - 4 \sin \theta \cos \theta = 4.$$

Dividing by $\cos^2 \theta$ and using $\tan \theta = \frac{\sin \theta}{\cos \theta}$ gives

$$7 \tan^2 \theta - 4 \tan \theta = 4 \sec^2 \theta.$$

Since $\cos \theta = 0$ does not satisfy the original equation, this division loses no solutions. Using $\sec^2 \theta = 1 + \tan^2 \theta$,

$$7 \tan^2 \theta - 4 \tan \theta = 4 + 4 \tan^2 \theta \Rightarrow 3 \tan^2 \theta - 4 \tan \theta - 4 = 0.$$

((a) 4)

$$3 \tan^2 x - 4 \tan x - 4 = 0 \Rightarrow (3 \tan x + 2)(\tan x - 2) = 0.$$

$$\tan x = 2 \quad \text{or} \quad \tan x = -\frac{2}{3}.$$

In $0^\circ < x < 360^\circ$, this gives

$$x = 63.4^\circ, 146.3^\circ, 243.4^\circ, 326.3^\circ$$

((b) 4)

Put $u = 4\alpha$. The smallest u greater than 2880° satisfying the equation is $63.4349\dots^\circ + 8(360^\circ) = 2943.4349\dots^\circ$.

$$\alpha = \frac{2943.4349\dots^\circ}{4} = 735.8587\dots^\circ.$$

$$\alpha = 735.9^\circ$$

((c) 1)

Question 14

Question 14

Prove, using algebra, that $n^2 + 5n$ is even for all $n \in \mathbb{N}$.

Model solution

Every natural number is either even or odd.

If $n = 2k$, where $k \in \mathbb{N}$, then

$$n^2 + 5n = (2k)^2 + 5(2k) = 4k^2 + 10k = 2(2k^2 + 5k),$$

which is even.

If $n = 2k + 1$, where $k \in \mathbb{N} \cup \{0\}$, then

$$n^2 + 5n = (2k + 1)^2 + 5(2k + 1) = 4k^2 + 14k + 6 = 2(2k^2 + 7k + 3),$$

which is even.

Since these cases cover every $n \in \mathbb{N}$,

$$n^2 + 5n \text{ is even for all } n \in \mathbb{N}.$$

(4)