



Edexcel AS Mathematics

8MA0/22 - Mechanics

Thursday 25 May 2023

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

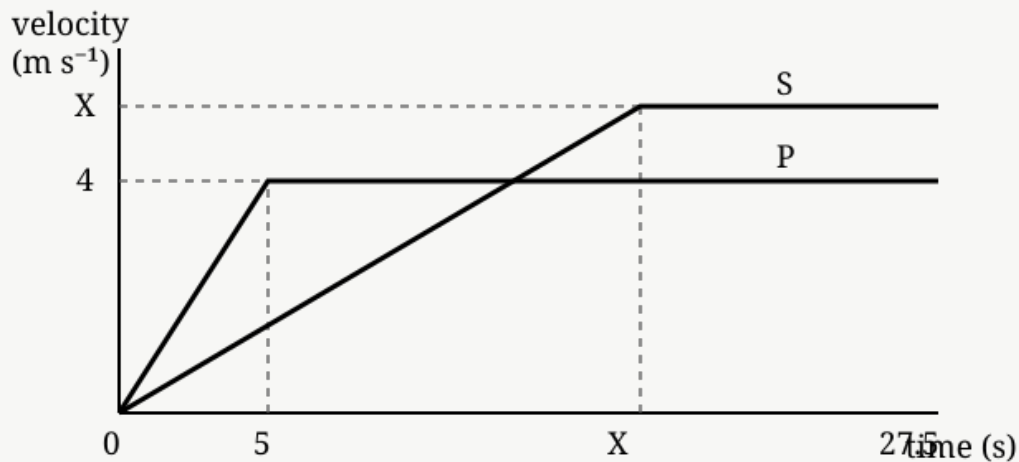
Contents

Question 1	2
Question 2	4
Question 3	5
Question 4	7

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Question 1

Question 1



Two children, Pat (P) and Sam (S), run a race along a straight horizontal track. Both start from rest at the same time and cross the finish line together, 27.5 s later. The velocity–time graphs above describe the following model.

Pat accelerates uniformly from rest for 5 s to 4 m s^{-1} , then continues at 4 m s^{-1} . Sam accelerates uniformly at 1 m s^{-2} from rest to $X \text{ m s}^{-1}$, then continues at $X \text{ m s}^{-1}$.

- Explain why the areas under the two graphs are equal. (1)
- Find Pat's acceleration during the first 5 s. (1)
- Find the length of the race. (2)
- Find X , giving your answer to 3 significant figures. (4)

Model solution

(a) The area under a velocity–time graph is the displacement. Pat and Sam each travel from the same start to the same finish line, so their displacements, and hence the areas, are equal.

The areas are equal because both children travel the same distance.

(b) For Pat,

$$a = \frac{4 - 0}{5} = 0.8.$$

$$0.8 \text{ m s}^{-2}$$

(c) The race length is the area under Pat's graph:

$$\frac{1}{2}(5)(4) + (27.5 - 5)(4) = 10 + 90 = 100.$$

$$100 \text{ m}$$

(d) Sam accelerates at 1 m s^{-2} from rest, so the time taken to reach speed X is

$$\frac{X}{1} = X \text{ s.}$$

Equating the area under Sam's graph to the race length gives

$$\frac{1}{2}X \cdot X + (27.5 - X)X = 100.$$

Thus

$$\frac{1}{2}X^2 - 27.5X + 100 = 0,$$

so

$$X^2 - 55X + 200 = 0.$$

Therefore

$$X = \frac{55 \pm \sqrt{55^2 - 4(200)}}{2} = \frac{55 \pm \sqrt{2225}}{2}.$$

The root $\frac{55 + \sqrt{2225}}{2} \approx 51.1$ is not possible, since Sam cannot spend 51.1 s accelerating during a 27.5 s race. Hence

$$X = \frac{55 - \sqrt{2225}}{2} \approx 3.915.$$

$$X = 3.92 \text{ m s}^{-1} \text{ (3 s.f.)}$$

For each child, displacement is represented by the area between their velocity--time graph and the time axis.

Question 2

Question 2

A small stone is projected vertically upwards with speed 39.2 m s^{-1} from a point O. It is modelled as a particle moving freely under gravity until it hits the ground 10 s later.

- (a) Find the height of O above the ground. (3)
- (b) Find the total length of time for which the speed of the stone is less than or equal to 24.5 m s^{-1} . (3)
- (c) State one refinement to the model that would make the answer to part (a) more accurate. (1)

Model solution

Take vertically upwards as positive. Then $u = 39.2 \text{ m s}^{-1}$ and $a = -9.8 \text{ m s}^{-2}$.

(a) Let the displacement from O to the ground be s . At $t = 10$,

$$s = ut + \frac{1}{2}at^2 = 39.2(10) + \frac{1}{2}(-9.8)(10)^2 = -98.$$

Thus the ground is 98 m below O.

98 m

(b) The velocity at time t is

$$v = 39.2 - 9.8t.$$

The speed is at most 24.5 m s^{-1} between the two times at which $v = 24.5$ and $v = -24.5$:

$$39.2 - 9.8t = 24.5 \quad \Rightarrow \quad t = 1.5,$$

$$39.2 - 9.8t = -24.5 \quad \Rightarrow \quad t = 6.5.$$

Therefore the required total time is

$$6.5 - 1.5 = 5.$$

5 s

(c) Include air resistance in the model.

For example, allow for air resistance.

The stone's speed is the magnitude of its velocity, so both $v = 24.5$ and $v = -24.5$ must be considered.

Question 3

Question 3

In this question, show all stages of your working; solutions relying entirely on calculator technology are not acceptable.

A fixed point O lies on a straight line. A particle P moves along the line. At time t s, where $t \geq 0$, after passing through O, its velocity is modelled by

$$v = 15 - t^2 - 2t \text{ m s}^{-1}.$$

- (a) Verify that P comes to instantaneous rest when $t = 3$. (1)
- (b) Find the magnitude of the acceleration of P when $t = 3$. (3)
- (c) Find the total distance travelled in $0 \leq t \leq 4$. Show all stages of working. (4)

Model solution

(a) Substituting $t = 3$ into the velocity model gives

$$v = 15 - 3^2 - 2(3) = 15 - 9 - 6 = 0.$$

Therefore P is instantaneously at rest when $t = 3$.

$$v = 0 \text{ m s}^{-1} \text{ when } t = 3.$$

(b) Acceleration is the derivative of velocity:

$$a = \frac{dv}{dt} = -2t - 2.$$

When $t = 3$,

$$a = -2(3) - 2 = -8 \text{ m s}^{-2}.$$

Hence its magnitude is

$$8 \text{ m s}^{-2}$$

(c) An antiderivative of the velocity is

$$\int (15 - t^2 - 2t) dt = 15t - \frac{1}{3}t^3 - t^2.$$

Since

$$v = 15 - t^2 - 2t = (3 - t)(t + 5),$$

the velocity is positive for $0 \leq t < 3$ and negative for $3 < t \leq 4$. Thus P changes direction at $t = 3$.

The distance travelled from $t = 0$ to $t = 3$ is

$$\left[15t - \frac{1}{3}t^3 - t^2 \right]_0^3 = 45 - 9 - 9 = 27 \text{ m}.$$

Its displacement from $t = 3$ to $t = 4$ is

$$\left[15t - \frac{1}{3}t^3 - t^2 \right]_3^4 = \left(60 - \frac{64}{3} - 16 \right) - 27 = -\frac{13}{3} \text{ m}.$$

Therefore the distance in this latter interval is $\frac{13}{3}$ m, and the total distance is

$$27 + \frac{13}{3} = \frac{94}{3}.$$

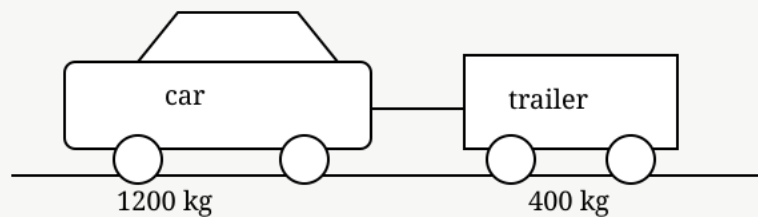
$$\frac{94}{3} \text{ m}$$

The negative displacement after $t = 3$ indicates a reversal of direction, so its magnitude must be added when finding total distance.

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Question 4

Question 4



A car of mass 1200 kg tows a trailer of mass 400 kg along a straight horizontal road using a horizontal tow rope parallel to the motion. The driving force of the car is 7400 N and the tension in the rope is 2400 N.

The resistance to motion of the car is modelled as a constant force of magnitude $2R$ N, and that of the trailer as a constant force of magnitude R N. The rope is modelled as light and inextensible. The acceleration of the car is $a \text{ m s}^{-2}$.

(a) Find a . (5)

In a refined model, the rope has mass and the acceleration of the car is $a_1 \text{ m s}^{-2}$.

(b) State how a_1 compares with a . (1)

(c) State one limitation of the model for the resistance to motion of the car. (1)

Model solution

Take the direction of motion as positive. Since the rope is light and inextensible, the car and trailer have the same acceleration a .

(a) Applying Newton's second law to the car gives

$$7400 - 2400 - 2R = 1200a.$$

Applying Newton's second law to the trailer gives

$$2400 - R = 400a.$$

From the trailer equation,

$$R = 2400 - 400a.$$

Substituting this into the car equation,

$$5000 - 2(2400 - 400a) = 1200a,$$

so

$$200 + 800a = 1200a,$$

and hence

$$a = 0.5.$$

$$a = 0.5 \text{ m s}^{-2}$$

(b) A rope with mass must also be accelerated. With the same driving force, the acceleration of the car is therefore smaller.

$$a_1 < a$$

(c) The resistance to motion of a car is not constant in reality; for example, air resistance changes with speed.

The resistance force is not constant.

The tension pulls forwards on the trailer and backwards on the car; resistances act opposite to the direction of motion.

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