



Edexcel AQA AS Mathematics

7356/1 - Pure Mathematics and Mechanics

Thursday 18 May 2023

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 2	3
Question 3	4
Question 4	5
Question 5	6
Question 6	7
Question 7	8
Question 8	9
Question 9	10
Question 10	12
Question 11	13
Question 12	14
Question 13	15
Question 14	16
Question 15	17
Question 16	18
Question 17	19
Question 18	20

Question 1

Question 1

At a point P on a curve, the gradient of the tangent is 10. State the gradient of the normal at P.

Model solution

A normal is perpendicular to the tangent, so its gradient is the negative reciprocal of the tangent gradient.

$$m_{\text{normal}} = -\frac{1}{10} = -0.1.$$

−0.1

(1)

www.mathswithmatt.co.uk

Question 2

Question 2

Identify the expression equivalent to $\left(\frac{2x}{5}\right)^{-3}$.

Model solution

A negative power means take the reciprocal before cubing.

$$\left(\frac{2x}{5}\right)^{-3} = \left(\frac{5}{2x}\right)^3 = \frac{125}{8x^3}.$$

$$\frac{125}{8x^3}$$

(1)

www.mathswithmatt.co.uk

Question 3

Question 3

The coefficient of x^2 in the binomial expansion of $(1 + ax)^6$ is 20. Find the two possible values of a .

Model solution

The x^2 term is obtained by choosing $(ax)^2$ in the binomial expansion.

$$\binom{6}{2}(ax)^2 = 15a^2x^2.$$

Equate its coefficient to the given coefficient.

$$15a^2 = 20 \implies a^2 = \frac{4}{3} \implies a = \pm \frac{2}{\sqrt{3}} = \pm \frac{2\sqrt{3}}{3}.$$

$$a = \pm \frac{2\sqrt{3}}{3}$$

(3)

Question 4

Question 4

Given that $5 \cos^2 y - 4 \sin^2 y = 0$: (a) find the possible values of $\tan y$ in exact form; (b) solve the equation for $0^\circ \leq y \leq 360^\circ$, to the nearest 0.1° .

Model solution

(a) The equation contains squared sine and cosine, so dividing by $\cos^2 y$ produces $\tan^2 y$.

$$5 \cos^2 y = 4 \sin^2 y \implies 5 = 4 \frac{\sin^2 y}{\cos^2 y} = 4 \tan^2 y.$$

$$\tan^2 y = \frac{5}{4} \implies \tan y = \pm \frac{\sqrt{5}}{2}.$$

$$\tan y = \pm \frac{\sqrt{5}}{2}$$

(3) (b) The reference angle is found using the positive tangent value. Tangent is positive in quadrants I and III, and negative in II and IV.

$$\alpha = \tan^{-1} \left(\frac{\sqrt{5}}{2} \right) = 48.189 \dots^\circ.$$

$$y = 48.2^\circ, 131.8^\circ, 228.2^\circ, 311.8^\circ.$$

$$48.2^\circ, 131.8^\circ, 228.2^\circ, 311.8^\circ$$

(2)

Question 5

Question 5

- (a) Given $y = x\sqrt{x}$, find $\frac{dy}{dx}$. (b) The line $6x - 2y + 5 = 0$ is a tangent to $y = x\sqrt{x} + k$. Find k .

Model solution

(a) Write the square root as a fractional power so that the power rule can be used.

$$y = x^{3/2} \implies \frac{dy}{dx} = \frac{3}{2}x^{1/2} = \frac{3}{2}\sqrt{x}.$$

$$\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$$

(2) (b) At a point of contact, the curve and tangent have the same gradient. First rearrange the line to identify its gradient.

$$2y = 6x + 5 \implies y = 3x + \frac{5}{2}, \quad m_{\text{line}} = 3.$$

$$\frac{3}{2}\sqrt{x} = 3 \implies \sqrt{x} = 2 \implies x = 4.$$

The line gives the y -coordinate of the contact point. Substitution into the curve then determines its vertical shift.

$$y = 3(4) + \frac{5}{2} = \frac{29}{2}, \quad \frac{29}{2} = 4\sqrt{4} + k = 8 + k.$$

$$k = \frac{13}{2}.$$

$$k = \frac{13}{2}$$

(5)

Question 6

Question 6

The curve C_1 has equation $y = 2x^2 - 20x + 42$. (a) Express it as $y = a(x - b)^2 + c$. (b) Write down its minimum point. (c) C_1 is mapped to C_2 by a stretch in the y -direction; the minimum of C_2 is $(5, -4)$. Find C_2 .

Model solution

(a) Complete the square; factor out 2 first so the coefficient of x^2 inside the bracket is 1.

$$y = 2(x^2 - 10x + 21) = 2((x - 5)^2 - 25 + 21) = 2(x - 5)^2 - 8.$$

$$y = 2(x - 5)^2 - 8$$

(3) (b) In completed-square form, the squared term is smallest when it is zero.

$$x = 5 \implies y = -8.$$

$$(5, -8)$$

(1) (c) A vertical stretch leaves x unchanged and multiplies all y -coordinates by its scale factor:

$$\text{scale factor} = \frac{-4}{-8} = \frac{1}{2}.$$

$$y = \frac{1}{2}(2(x - 5)^2 - 8) = (x - 5)^2 - 4 = x^2 - 10x + 21.$$

$$y = (x - 5)^2 - 4$$

(2)

Question 7

Question 7

Points P and Q lie on $y = x^4$, with x-coordinates x and $x + h$ respectively. (a) Expand $(x + h)^4$. (b) Hence find the gradient of PQ. (c) Explain how to obtain the gradient function of $y = x^4$.

Model solution

(a) Apply the binomial theorem with coefficients 1, 4, 6, 4, 1.

$$(x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4.$$

$$x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

(2) (b) A secant gradient is change in y divided by change in x .

$$\frac{(x + h)^4 - x^4}{(x + h) - x} = \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} = 4x^3 + 6x^2h + 4xh^2 + h^3.$$

$$4x^3 + 6x^2h + 4xh^2 + h^3$$

(1) (c) The tangent gradient is the limiting value of the secant gradient as Q approaches P, so let $h \rightarrow 0$.

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} (4x^3 + 6x^2h + 4xh^2 + h^3) = 4x^3.$$

$$\frac{dy}{dx} = 4x^3$$

(2)

Question 8

Question 8

(a) Show that $\int_1^a \left(6 - \frac{12}{\sqrt{x}}\right) dx = 6a - 24\sqrt{a} + 18$. (b) The curve $y = 6 - \frac{12}{\sqrt{x}}$, the lines $x = 1$, $x = a$ and the x -axis form equal shaded regions R_1 and R_2 . Find a , fully justifying your answer.

Model solution

(a) Rewrite the reciprocal square root as $x^{-1/2}$, then integrate term by term.

$$\int \left(6 - 12x^{-1/2}\right) dx = 6x - 24x^{1/2}.$$

$$\left[6x - 24\sqrt{x}\right]_1^a = (6a - 24\sqrt{a}) - (6 - 24) = 6a - 24\sqrt{a} + 18.$$

$$6a - 24\sqrt{a} + 18$$

(3) (b) The curve meets the axis when $6 - 12/\sqrt{x} = 0$, giving $x = 4$. It is below the axis from $x = 1$ to 4 and above it after 4. Equal areas therefore mean that the signed area from 1 to a is zero.

$$6a - 24\sqrt{a} + 18 = 0 \implies a - 4\sqrt{a} + 3 = 0.$$

Use $u = \sqrt{a}$ because both a and $\sqrt{a}$ occur.

$$u^2 - 4u + 3 = 0 \implies (u - 1)(u - 3) = 0.$$

$$\sqrt{a} = 1 \text{ or } 3 \implies a = 1 \text{ or } 9.$$

$a = 1$ gives no second region, so it cannot represent the equal non-zero shaded areas.

$$a = 9$$

(4)

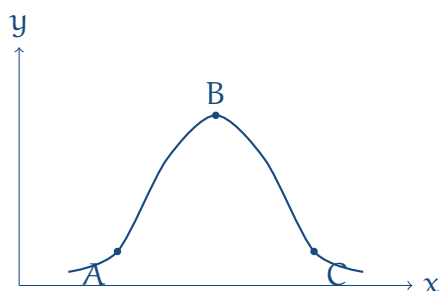
Question 9

Question 9

A continuous curve $y = f(x)$ passes through $A(2,1)$, $B(4,5)$ and $C(6,1)$, and $f'(4) = 0$. Sketch a curve for which: (a) both “there is a turning point at B” and “B is a maximum” are correct; (b) the first is correct and the second is not; (c) neither is correct.

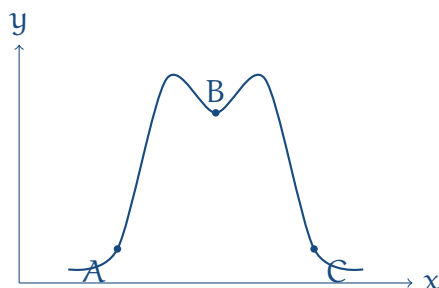
Model solution

Since $f'(4) = 0$, every sketch must have a horizontal tangent at B. The stated nature is decided by whether the curve changes direction there. A valid sketch for (a) is:



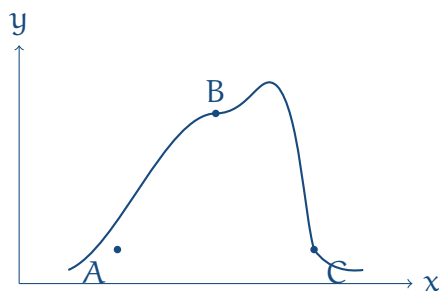
(a) A curve through A, B, C with a local maximum at B.

(1) For a turning point that is not a maximum, make B a local minimum. A valid sketch for (b) is:



(b) A curve through A, B, C with a local minimum at B.

(1) For a stationary point that is not a turning point, keep the curve increasing through its horizontal tangent at B, then allow it to turn later in order to reach C. A valid sketch for (c) is:



(c) A curve through A, B, C with a stationary point of inflection at B.

(1)

www.mathswithmatt.co.uk

Question 10

Question 10

A car is worth £18000 on 1 January 2016 and £12000 on 1 January 2018; its value decreases exponentially. (a) Decide whether Charlie's prediction of £6000 after another two years or Kaya's prediction of £8000 is correct. (b) $V = Ae^{-kt}$, where t is years after 1 January 2016. Find t when $V = 10000$, to two significant figures. (c) Explain why the model is unsuitable at $t = 30$.

Model solution

(a) Exponential decay applies the same multiplier, not the same cash decrease, in equal time intervals.

$$\frac{12000}{18000} = \frac{2}{3}, \quad \frac{2}{3} \times 12000 = 8000.$$

Kaya is correct: the value after a further two years is £8000.

(2) (b) At $t = 0$, the model gives $V = A$, so the purchase value determines A . Use the value after two years to find k .

$$A = 18000, \quad 12000 = 18000e^{-2k}.$$

$$e^{-2k} = \frac{2}{3} \implies k = \frac{1}{2} \ln\left(\frac{3}{2}\right) = 0.2027 \dots$$

Now substitute the target value and solve using logarithms.

$$10000 = 18000e^{-0.2027 \dots t} \implies t = \frac{\ln(1.8)}{0.2027 \dots} = 2.901 \dots$$

$$t = 2.9 \text{ years}$$

(5) (c) Long-term exponential depreciation predicts an unrealistically tiny value and ignores a car's scrap value or likely disposal.

The car would probably have been scrapped, so the model is not realistic at $t = 30$.

(1)

Question 11

Question 11

(a) A circle has equation $x^2 + y^2 - 10x - 6 = 0$. Find its centre and radius. (b) An equilateral triangle has one vertex at the origin and a vertical side on $x = 8$. (i) Show the origin is inside the circle. (ii) Prove the whole triangle lies inside the circle.

Model solution

(a) Complete the square in x to obtain centre--radius form.

$$x^2 - 10x + y^2 = 6 \implies (x - 5)^2 + y^2 = 31.$$

Centre (5,0); radius $\sqrt{31}$.

(2) (b)(i) Compare the distance of the origin from the centre with the radius.

$$\sqrt{(0-5)^2 + 0^2} = 5 < \sqrt{31}.$$

The origin lies inside the circle.

(1) (b)(ii) The perpendicular distance from the origin to the vertical side is 8. Splitting the equilateral triangle into two right triangles gives the half-side height using $\tan 30^\circ$.

$$\tan 30^\circ = \frac{y}{8} \implies y = 8 \tan 30^\circ = \frac{8}{\sqrt{3}}.$$

Thus the other two vertices are $(8, \pm 8/\sqrt{3})$. Their distance from the circle's centre is

$$\sqrt{(8-5)^2 + \left(\frac{8}{\sqrt{3}}\right)^2} = \sqrt{9 + \frac{64}{3}} = \sqrt{\frac{91}{3}} < \sqrt{31}.$$

Both are inside; by symmetry this checks both vertices. A circle is convex, so every line segment joining points inside it is inside it. Hence all three sides, and therefore the triangle, lie completely within the circle.

The triangle lies completely within the circle.

(4)

Question 12

Question 12

A particle starts from rest and accelerates forward at a constant $a \text{ m s}^{-2}$. After 6 seconds its velocity is 3 m s^{-1} . Find a .

Model solution

Use $v = u + at$ with initial velocity $u = 0$.

$$3 = 0 + 6a \implies a = 0.5 \text{ m s}^{-2}.$$

$$0.5 \text{ m s}^{-2}$$

(1)

www.mathswithmatt.co.uk

Question 13

Question 13

A resultant force $\begin{pmatrix} -2 \\ 6 \end{pmatrix}$ N acts on a particle whose acceleration is $\begin{pmatrix} -6 \\ y \end{pmatrix}$ m s⁻². Find y .

Model solution

Force and acceleration have the same direction because $F = ma$ for positive mass. Their corresponding components are in a constant ratio.

$$\frac{-2}{-6} = \frac{6}{y} \implies \frac{1}{3} = \frac{6}{y} \implies y = 18.$$

18

(1)

Question 14

Question 14

A ball is dropped from rest from height h . After 4 seconds its height is $0.2h$. (a) Ignoring air resistance, show $h = 10g$. (b) Explain the effect on h if air resistance cannot be ignored.

Model solution

(a) Take downward as positive. In four seconds the ball has fallen $h - 0.2h = 0.8h$; it starts from rest and has acceleration g .

$$s = ut + \frac{1}{2}at^2 \implies 0.8h = 0 + \frac{1}{2}g(4)^2 = 8g.$$

$$h = 10g.$$

$$h = 10g$$

(3) (b) Air resistance acts upward, reducing the downward acceleration and hence the distance fallen in four seconds for a given h . To fall $0.8h$ in that time, h must be smaller.

h would be less than $10g$.

(1)

Question 15

Question 15

A velocity–time graph is a straight line from $(0, -4)$ to $(4, 10)$, then horizontal at $v = 10$ until $t = 9$.
 (a) Show the acceleration over the first 4 seconds is 3.5 m s^{-2} . (b) The particle starts at P. Show its displacement from P at $t = 9$ is 62 m.

Model solution

(a) Acceleration is the gradient of a velocity--time graph.

$$a = \frac{10 - (-4)}{4 - 0} = \frac{14}{4} = 3.5 \text{ m s}^{-2}.$$

$$3.5 \text{ m s}^{-2}$$

(1) (b) Displacement is signed area under the velocity--time graph. The graph crosses the time axis when $-4 + 3.5t = 0$, so $t = 8/7$.

$$s_{0 \rightarrow 4} = -\frac{1}{2} \left(\frac{8}{7} \right) (4) + \frac{1}{2} \left(4 - \frac{8}{7} \right) (10) = -\frac{16}{7} + \frac{100}{7} = 12.$$

The remaining rectangle has area $10(9 - 4) = 50$.

$$s_{0 \rightarrow 9} = 12 + 50 = 62 \text{ m}.$$

$$62 \text{ m}$$

(3)

Question 16

Question 16

A speed boat has velocity $v = 0.9 + 0.16t - 0.06t^2 \text{ m s}^{-1}$, t seconds after launch. (a) Find its acceleration at $t = 2$. (b) Find its displacement from launch at $t = 2$.

Model solution

(a) Acceleration is the rate of change of velocity, so differentiate with respect to time.

$$a = \frac{dv}{dt} = 0.16 - 0.12t, \quad a(2) = 0.16 - 0.24 = -0.08 \text{ m s}^{-2}.$$

$$-0.08 \text{ m s}^{-2}$$

(3) (b) Displacement is the integral of velocity. The launch point means $s = 0$ when $t = 0$.

$$s = \int (0.9 + 0.16t - 0.06t^2) dt = 0.9t + 0.08t^2 - 0.02t^3 + C.$$

$$0 = s(0) \implies C = 0, \quad s(2) = 0.9(2) + 0.08(2)^2 - 0.02(2)^3 = 1.96.$$

$$1.96 \text{ m}$$

(4)

Question 17

Question 17

A particle on a smooth horizontal surface has resultant force $\begin{pmatrix} 12 \\ 9 \end{pmatrix}$ N and moves in a straight line.

(a) Find its magnitude. (b) $\vec{OA} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ m and $\vec{OB} = \begin{pmatrix} k \\ k-1 \end{pmatrix}$ m; the particle moves parallel to $\vec{AB}$.

(i) Find $\vec{AB}$. (ii) Find k .

Model solution

(a) Use Pythagoras on the perpendicular vector components.

$$|\mathbf{F}| = \sqrt{12^2 + 9^2} = 15 \text{ N.}$$

$$15 \text{ N}$$

(1) (b)(i) A displacement vector is final position vector minus initial position vector.

$$\vec{AB} = \begin{pmatrix} k \\ k-1 \end{pmatrix} - \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} k-3 \\ k-8 \end{pmatrix} \text{ m.}$$

$$\begin{pmatrix} k-3 \\ k-8 \end{pmatrix} \text{ m}$$

(1) (b)(ii) On a smooth surface, the resultant force gives the direction of acceleration and hence, from rest, the direction of motion. Parallel vectors are scalar multiples.

$$\begin{pmatrix} k-3 \\ k-8 \end{pmatrix} = \lambda \begin{pmatrix} 12 \\ 9 \end{pmatrix}.$$

$$9(k-3) = 12(k-8) \implies 9k - 27 = 12k - 96 \implies k = 23.$$

$$k = 23$$

(2)

Question 18

Question 18

A 2780 kg rescue van tows a 1620 kg car on a horizontal road at 0.6 m s^{-2} . The van has driving force D and resistance R ; the car has resistance $0.6R$. The tow-bar tension is $T = kD - 18$. (a) Find k . (b) State one assumption required.

Model solution

(a) Take the direction of travel as positive. Apply $F = ma$ separately: tension pulls the car forward, while it pulls the van backward.

$$\text{van: } D - R - T = 2780(0.6) = 1668,$$

$$\text{car: } T - 0.6R = 1620(0.6) = 972.$$

Eliminate R . From the combined system, $D - 1.6R = 2640$, so $0.6D - 0.96R = 1584$. Multiply the car equation by 1.6 and substitute for $0.96R$.

$$1.6T - 0.96R = 1555.2,$$

$$0.6D - 1.6T = 28.8 \implies T = \frac{0.6}{1.6}(D - 48) = \frac{3}{8}D - 18.$$

Comparing this with $T = kD - 18$ gives

$$k = \frac{3}{8}$$

(5) (b) One modelling assumption about the geometry of the towing arrangement is needed.

The tow bar is horizontal.

(1)