



Edexcel AQA AS Mathematics

7356/1 - Pure Mathematics and Mechanics

Thursday 16 May 2024

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2024 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Given $\tan \theta^\circ = k$, find $\tan(\theta + 180)^\circ$.

Model solution

Tangent has period 180° , so adding 180° does not change its value.

$$\tan(\theta + 180)^\circ = \tan \theta^\circ = k.$$

k

(1)

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Question 2

Question 2

Curve C has equation $y = \frac{1}{(x-1)^2}$. State the equations of its asymptotes.

Model solution

The denominator is zero at $x = 1$, giving a vertical asymptote. As $|x|$ grows, the fraction tends to zero.

$$x = 1 \text{ and } y = 0$$

(1)

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Question 3

Question 3

Express $\frac{\sqrt{3} + 3\sqrt{5}}{\sqrt{5} - \sqrt{3}}$ in the form $a + \sqrt{b}$, where a and b are integers. Fully justify your answer.

Model solution

Multiply by the conjugate of the denominator to rationalise it.

$$\frac{\sqrt{3} + 3\sqrt{5}}{\sqrt{5} - \sqrt{3}} = \frac{(\sqrt{3} + 3\sqrt{5})(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} = \frac{18 + 4\sqrt{15}}{2} = 9 + 2\sqrt{15}.$$

Since $2\sqrt{15} = \sqrt{60}$, this is in the required form.

$$9 + \sqrt{60}$$

(4)

Question 4

Question 4

- (a)(i) Show that $\sin \theta \tan \theta = 4 \cos \theta$ can be written as $\tan^2 \theta = 4$.
 (a)(ii) Hence solve the equation for $0^\circ < \theta < 360^\circ$.
 (b) Deduce all solutions of $\sin 3\alpha \tan 3\alpha = 4 \cos 3\alpha$ for $0^\circ < \alpha < 180^\circ$.

Model solution

(a)(i) Use $\tan \theta = \sin \theta / \cos \theta$ to convert the equation into tangent.

$$\frac{\sin^2 \theta}{\cos \theta} = 4 \cos \theta \implies \frac{\sin^2 \theta}{\cos^2 \theta} = 4 \implies \tan^2 \theta = 4.$$

$$\tan^2 \theta = 4$$

(1)

(a)(ii) $\tan \theta = \pm 2$. The reference angle is $\tan^{-1} 2 = 63.435 \dots^\circ$; use the signs of tangent in all four quadrants.

$$\theta = 63^\circ, 117^\circ, 243^\circ, 297^\circ$$

(3)

(b) Put $\theta = 3\alpha$. Here $0^\circ < 3\alpha < 540^\circ$, so include the four angles above and the first two after adding 360° , then divide by 3.

$$3\alpha = 63.435 \dots, 116.565 \dots, 243.435 \dots, 296.565 \dots, 423.435 \dots, 476.565 \dots^\circ.$$

$$\alpha = 21^\circ, 39^\circ, 81^\circ, 99^\circ, 141^\circ, 159^\circ$$

(3)

Question 5

Question 5

A student tests whether $(x - 2)$ is a factor of $f(x)$ by calculating $f(-2)$, finding $f(-2) = 0$. (a) Comment on the method. (b) Make two comments about their conclusion that $(x - 2)$ is a factor.

Model solution

(a) The factor theorem says $(x - a)$ is a factor exactly when $f(a) = 0$.

They should calculate $f(2)$, not $f(-2)$.

(1) (b) The calculation tests a different factor, so it cannot settle the suggested one.

$(x - 2)$ may or may not be a factor; $f(-2) = 0$ does show that $(x + 2)$ is a factor.

(2)

Question 6

Question 6

Determine, in exact set notation, the values of x satisfying $3x^2 + 3x > x + 6$.

Model solution

Rearrange to one quadratic, find its roots, then use that a positive-leading-coefficient quadratic is positive outside its roots.

$$3x^2 + 2x - 6 > 0, \quad x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-6)}}{6} = \frac{-1 \pm \sqrt{19}}{3}.$$

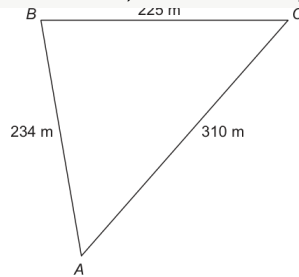
$$\left\{ x : x < \frac{-1 - \sqrt{19}}{3} \right\} \cup \left\{ x : x > \frac{-1 + \sqrt{19}}{3} \right\}$$

(4)

Question 7

Question 7

In triangular field ABC, $AB = 234$ m, $BC = 225$ m and $AC = 310$ m.



(a) Find angle A

(a) Find angle A. (b) One sheep requires 1200 m^2 of grass. Find the maximum number of sheep.

Model solution

(a) All three sides are known, so use the cosine rule for angle A, whose opposite side has length 225.

$$225^2 = 234^2 + 310^2 - 2(234)(310) \cos A, \quad \cos A = 0.690867 \dots$$

$$A = 46.3^\circ$$

(2) (b) Two sides and their included angle are known, so use the triangle area formula. Divide by the area per sheep and round down because only whole sheep are possible.

$$\text{Area} = \frac{1}{2}(234)(310) \sin(46.3 \dots^\circ) = 26223 \dots \text{ m}^2, \quad \frac{26223 \dots}{1200} = 21.85 \dots$$

21 sheep

(3)

Question 8

Question 8

Given $\ln x - \ln y = 3$: (a) express x in terms of y without logarithms; (b) given $x + y = 10$, find exact x and y .

Model solution

(a) Combine the logarithms with the quotient law, then exponentiate.

$$\ln\left(\frac{x}{y}\right) = 3 \implies \frac{x}{y} = e^3 \implies x = e^3 y.$$

$$x = e^3 y$$

(3) (b) Substitute this expression into the second equation to leave one unknown.

$$e^3 y + y = 10 \implies y = \frac{10}{e^3 + 1}, \quad x = \frac{10e^3}{e^3 + 1}.$$

$$y = \frac{10}{e^3 + 1}, \quad x = \frac{10e^3}{e^3 + 1}$$

(3)

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Question 9

Question 9

For $f(x) = x(6 - x)$: (a) find $f'(x)$; (b) sketch the gradient function, stating its intercepts.

Model solution

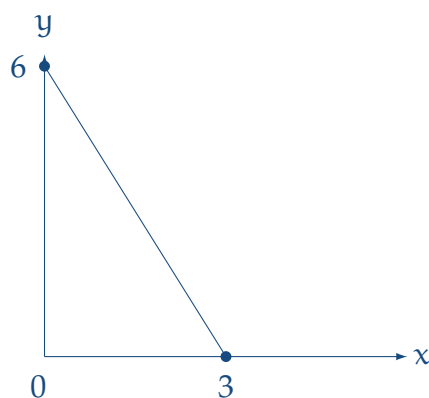
(a) Expand first, then differentiate each term.

$$f(x) = 6x - x^2 \implies f'(x) = 6 - 2x.$$

$$f'(x) = 6 - 2x$$

(2) (b) The gradient function is the line $y = 6 - 2x$. Its intercepts follow by setting $x = 0$ and $y = 0$ respectively.

$$(0, 6), \quad 6 - 2x = 0 \implies (3, 0).$$



A decreasing straight line through $(0, 6)$ and $(3, 0)$.

(3)

Question 10

Question 10

Given $\frac{dy}{dx} = (x+2)(2x-1)^2$ and $y = 900$ when $x = 6$, find y in terms of x .

Model solution

Expand before integrating, then use the given point to find the integration constant.

$$\frac{dy}{dx} = (x+2)(4x^2 - 4x + 1) = 4x^3 + 4x^2 - 7x + 2,$$

$$y = x^4 + \frac{4}{3}x^3 - \frac{7}{2}x^2 + 2x + c.$$

$$900 = 1296 + 288 - 126 + 12 + c \implies c = -570.$$

$$y = x^4 + \frac{4}{3}x^3 - \frac{7}{2}x^2 + 2x - 570$$

(6)

Question 11

Question 11

For continuous g , $g'(1) = 0$, $g'(4) = 0$ and $g''(x) = 2x - 5$. (a) Determine the nature of each turning point, fully justifying your answer. (b) Find where g is increasing.

Model solution

(a) The stationary points occur at $x = 1$ and $x = 4$. Apply the second derivative test: negative means local maximum and positive means local minimum.

$$g''(1) = -3 < 0, \quad g''(4) = 3 > 0.$$

Local maximum at $x = 1$; local minimum at $x = 4$.

(3) (b) The derivative is positive before the maximum, negative between the stationary points, and positive after the minimum.

$$x < 1 \text{ or } x > 4$$

(2)

Question 12

Question 12

The model $T = 15 + 8 \sin(30m - 120)^\circ$ gives monthly mean temperature T in $^\circ\text{C}$, where January is $m = 1$. (a) Find T for May. (b) Find the hottest month. (c)(i) State which parameter increases with an overall temperature rise, with a reason. (c)(ii) State which parameter increases with more extreme temperatures, with a reason.

Model solution

(a) May is month 5, so substitute $m = 5$.

$$T = 15 + 8 \sin(30(5) - 120)^\circ = 15 + 8 \sin 30^\circ = 19.$$

19 $^\circ\text{C}$

(2) (b) The temperature is greatest when the sine term is 1, which occurs when its angle is 90° .

$$30m - 120 = 90 \implies m = 7.$$

July

(2) (c)(i) The constant 15 is the vertical shift, so increasing it raises every monthly temperature.

Increase 15.

(1) (c)(ii) The coefficient 8 is the amplitude. Increasing it makes highs higher and lows lower, giving more extreme temperatures.

Increase 8.

(1)

Question 13

Question 13

A particle has velocity $v = 3 - 4t$, where t is in seconds. Deduce its constant acceleration.

Model solution

Acceleration is the rate of change of velocity.

$$a = \frac{dv}{dt} = \frac{d}{dt}(3 - 4t) = -4.$$

$$-4 \text{ m s}^{-2}$$

(1)

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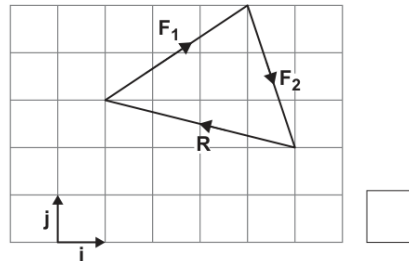
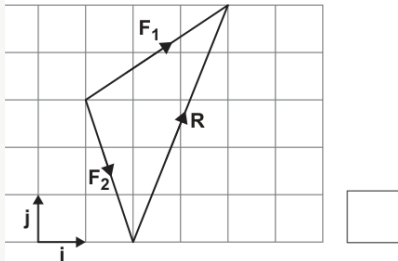
Question 14

Question 14

For $\mathbf{F}_1 = 3\mathbf{i} + 2\mathbf{j}$ N and $\mathbf{F}_2 = \mathbf{i} - 3\mathbf{j}$ N, identify the diagram representing $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$.

Tick (✓) **one** box.

[1 mark]



Model solution

Adding components gives the direction of the resultant, so select the matching head-to-tail vector diagram.

$$\mathbf{R} = (3 + 1)\mathbf{i} + (2 - 3)\mathbf{j} = 4\mathbf{i} - \mathbf{j}.$$

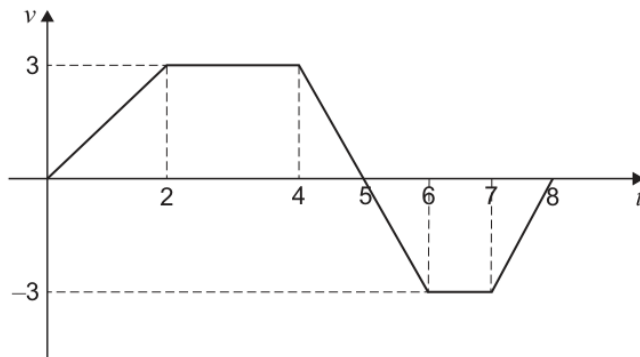
Bottom-left diagram

(1)

Question 15

Question 15

The velocity–time graph is shown. (a) Find total distance travelled in 8 seconds. (b) State when displacement is less than distance travelled.



Find the total distance travelled by the particle over the 8 second period.

Model solution

(a) Distance is total area between the graph and time axis, so count areas below the axis as positive.

$$\text{area above} = \frac{1}{2}(2)(3) + (2)(3) + \frac{1}{2}(1)(3) = 10.5,$$

$$\text{area below} = \frac{1}{2}(1)(3) + (1)(3) + \frac{1}{2}(1)(3) = 6.$$

$$10.5 + 6 = 16.5 \text{ m}$$

(3) (b) Before $t = 5$, velocity is non-negative, so distance equals displacement. After $t = 5$, the negative velocity represents reversal: distance adds the area but displacement subtracts it.

$$t > 5$$

(1)

Question 16

Question 16

Using $g = 9.8 \text{ m s}^{-2}$, a ball is launched vertically upwards with speed u and reaches maximum height 15 m. Ignore air resistance. Find u .

Model solution

Take upwards as positive. At maximum height $v = 0$, $a = -9.8 \text{ m s}^{-2}$ and $s = 15 \text{ m}$, so use $v^2 = u^2 + 2as$.

$$0 = u^2 + 2(-9.8)(15) \implies u^2 = 294, \quad u = 17.146\dots$$

Use the positive root because u is an upward launch speed.

$$u = 17 \text{ m s}^{-1}$$

(3)

Question 17

Question 17

A particle has acceleration $a = 10 - 6t$. Its velocity and displacement are initially zero. Show that $r = t^2(5 - t)$.

Model solution

Integrate acceleration to get velocity, using the initial velocity; then integrate velocity to get displacement, using the initial displacement.

$$v = \int (10 - 6t) \, dt = 10t - 3t^2 + c, \quad v(0) = 0 \implies c = 0.$$

$$r = \int (10t - 3t^2) \, dt = 5t^2 - t^3 + C, \quad r(0) = 0 \implies C = 0.$$

$$r = 5t^2 - t^3 = t^2(5 - t).$$

$$r = t^2(5 - t)$$

(4)

Question 18

Question 18

$\vec{OA} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ m and $\vec{OB} = \begin{pmatrix} 13 \\ 5 \end{pmatrix}$ m. (a) Show that $AB = 10$ m. (b) A 0.15 kg particle travels from A to B in 2 s, initially at 3 m s^{-1} towards B, under constant resultant force R. Find R.

Model solution

(a) Subtract the position vector of A from that of B, then find the magnitude of the displacement vector.

$$\vec{AB} = \begin{pmatrix} 13 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}, \quad AB = \sqrt{8^2 + 6^2} = 10 \text{ m}.$$

$$AB = 10 \text{ m}$$

(3) (b) Take the direction from A to B as positive. Use the known displacement, initial speed and time to find acceleration, then apply $F = ma$.

$$10 = 3(2) + \frac{1}{2}a(2^2) = 6 + 2a \implies a = 2 \text{ m s}^{-2},$$

$$R = 0.15 \times 2 = 0.3 \text{ N}.$$

$$R = 0.3 \text{ N}$$

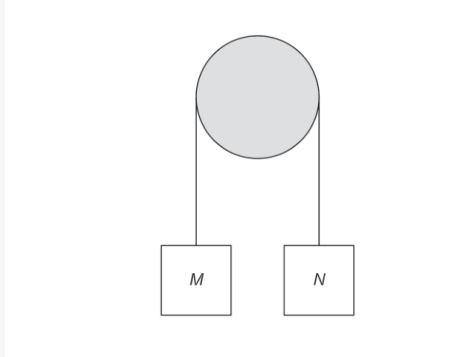
(3)

Question 19

Question 19

Masses $M = 0.6 \text{ kg}$ and $N = 0.5 \text{ kg}$ are connected by a light inextensible string over a smooth peg and released from rest.

nd N are initially held at rest, with the string taut, as shown in the



nd N are released at the same instant and begin to move vertical

(a) Show that $a = \frac{1}{11}g$. (b) After 0.5 s , N has speed $\frac{g}{k} \text{ m s}^{-1}$. Find k . (c) State one assumption required for part (b).

Model solution

(a) M is heavier, so take downward as positive for M and upward as positive for N . The taut inextensible string gives equal acceleration magnitudes. With tension T , use $F = ma$ on each mass.

$$0.6g - T = 0.6a \quad \text{for } M, \quad T - 0.5g = 0.5a \quad \text{for } N.$$

Adding the equations eliminates the internal tension.

$$0.1g = 1.1a \implies a = \frac{1}{11}g.$$

$$a = \frac{1}{11}g$$

(5) (b) The objects start from rest and accelerate constantly while the string remains taut, so use $v = u + at$.

$$v = 0 + \left(\frac{g}{11}\right)(0.5) = \frac{g}{22}.$$

$$k = 22$$

(2) (c) The motion used in part (b) must continue for the full 0.5 s .

N does not reach the peg.

(1)