



Edexcel AQA AS Mathematics

7356/2 - Pure Mathematics and Statistics

Wednesday 8 June 2022

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2022 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 2	3
Question 3	4
Question 4	5
Question 5	6
Question 6	7
Question 7	8
Question 8	9
Question 9	10
Question 10	12
Question 11	13
Question 12	14
Question 13	15
Question 14	16
Question 15	17
Question 16	18

Question 1

Question 1

Find $\int 12x^3 \, dx$.

(1)

Model solution

Use the reverse power rule: increase the power by one, then divide by the new power.

$$\int 12x^3 \, dx = 12 \left(\frac{x^4}{4} \right) + c = 3x^4 + c.$$

$$3x^4 + c.$$

www.mathswithmatt.co.uk

Question 2

Question 2

Given $\cos(y - 20^\circ) = \cos 60^\circ$, choose a possible value of y from $40^\circ, 140^\circ, 280^\circ, 320^\circ$. (1)

Model solution

Cosine has period 360° , and $\cos \theta = \cos 60^\circ$ when $\theta = \pm 60^\circ + 360^\circ n$. Test the listed values through the angle $y - 20^\circ$.

$$320^\circ - 20^\circ = 300^\circ = -60^\circ + 360^\circ,$$

so its cosine is $\cos 60^\circ$.

$$y = 320^\circ.$$

www.mathswithmatt.co.uk

Question 3

Question 3

A curve has equation $y = k\sqrt{x}$, where k is constant. Find $\frac{d^2y}{dx^2}$ at $(4, 2k)$, in terms of k . (5)

Model solution

Rewrite the square root as a fractional power so that the power rule can be applied twice. The x -coordinate of the stated point is 4, which is what is needed for the derivative.

$$y = kx^{1/2}, \quad \frac{dy}{dx} = \frac{k}{2}x^{-1/2}.$$

$$\frac{d^2y}{dx^2} = \frac{k}{2} \left(-\frac{1}{2}\right) x^{-3/2} = -\frac{k}{4}x^{-3/2}.$$

At $x = 4$, $4^{-3/2} = 1/(\sqrt{4})^3 = 1/8$. Hence

$$\left. \frac{d^2y}{dx^2} \right|_{x=4} = -\frac{k}{4} \cdot \frac{1}{8} = -\frac{k}{32}.$$

$$-\frac{k}{32}.$$

Question 4

Question 4

The equation $9x^2 + 4x + p^2 = 0$ has no real solutions for x . Find the possible values of p , fully justifying your answer. (4)

Model solution

A quadratic has no real roots precisely when its discriminant is negative. Here $a = 9$, $b = 4$, and $c = p^2$.

$$b^2 - 4ac < 0$$

$$4^2 - 4(9)(p^2) < 0$$

$$16 - 36p^2 < 0 \implies p^2 > \frac{4}{9}.$$

Taking both possible signs after solving the squared inequality gives the required two intervals.

$$p < -\frac{2}{3} \text{ or } p > \frac{2}{3}.$$

www.mathswithmatt.co.uk

Question 5

Question 5

$$f(x) = 2x^3 - 7x^2 - 12x + 45.$$

(a) Kaya states that $f(3) = 0$, and that this shows $(x + 3)$ is a factor of $f(x)$. State, with a reason, whether each statement is correct. (2)

(b) Fully factorise $f(x)$. (3)

Model solution

(a) Substitute $x = 3$ directly to check the first claim. The factor theorem says $f(r) = 0$ gives factor $(x - r)$, so the sign of the root matters.

$$f(3) = 2(3)^3 - 7(3)^2 - 12(3) + 45 = 54 - 63 - 36 + 45 = 0.$$

Thus statement 1 is correct. Statement 2 is incorrect: $f(3) = 0$ shows that $(x - 3)$, not $(x + 3)$, is a factor.

Statement 1 is correct; statement 2 is incorrect because the factor is $(x - 3)$.

(b) Since part (a) found the root 3, divide by $(x - 3)$ before factorising the remaining quadratic.

$$2x^3 - 7x^2 - 12x + 45 = (x - 3)(2x^2 - x - 15).$$

$$2x^2 - x - 15 = (x - 3)(2x + 5).$$

$$f(x) = (x - 3)^2(2x + 5).$$

Question 6

Question 6

Dog equivalent human age is $16 \ln(\text{dog age}) + 31$.

- (a) Find the equivalent age, to the nearest year, for a dog aged 5 years. (1)
- (b) A dog has equivalent age 40 years. Find its actual age to the nearest month. (3)
- (c) Explain why the behaviour of $\ln x$ close to zero means the formula cannot be true for very young dogs. (2)

Model solution

(a) The formula gives the human age directly once the dog age is substituted.

$$16 \ln 5 + 31 = 56.75 \dots$$

57 human years.

(b) Set the formula equal to 40. To undo $\ln$, exponentiate both sides.

$$16 \ln x + 31 = 40 \implies \ln x = \frac{9}{16}$$

$$x = e^{9/16} = 1.755 \dots \text{ years.}$$

$$1.755 \dots \times 12 = 21.06 \dots \text{ months.}$$

21 months (or 1 year 9 months).

(c) As $x \rightarrow 0^+$, $\ln x \rightarrow -\infty$. Therefore the formula predicts an arbitrarily negative equivalent human age for a sufficiently young positive dog age, which is not meaningful.

The model cannot be valid for very young dogs.

Question 7

Question 7

Write $\frac{3 - \sqrt{n}}{2 + \sqrt{n}}$ as $a + b\sqrt{n}$, where a, b, n are rational but $\sqrt{n}$ is irrational. Find a and b in terms of n . (4)

Model solution

Rationalise the denominator by multiplying by its conjugate $2 - \sqrt{n}$. This makes the denominator rational through the difference of two squares.

$$\frac{3 - \sqrt{n}}{2 + \sqrt{n}} \times \frac{2 - \sqrt{n}}{2 - \sqrt{n}} = \frac{6 - 3\sqrt{n} - 2\sqrt{n} + n}{4 - n} = \frac{6 + n - 5\sqrt{n}}{4 - n}.$$

Now split the two numerator terms to identify the rational part and the coefficient of $\sqrt{n}$.

$$\frac{6 + n}{4 - n} - \frac{5}{4 - n}\sqrt{n}.$$

$$a = \frac{6 + n}{4 - n}, \quad b = -\frac{5}{4 - n}.$$

Question 8

Question 8

Triangle ABC has side lengths $m - n$, m , and $m + n$, where $0 < 2n < m$, and angle A is the largest angle.

(a)(i) Explain why A is opposite the side of length $m + n$. (1)

(a)(ii) Using the cosine rule, show that $\cos A = \frac{m - 4n}{2(m - n)}$. (3)

(b) BC is the diameter of a circle and A lies on its circumference. Given $m = 8$, calculate n . (3)

Model solution

(a)(i) Since $n > 0$, $m + n$ is the greatest of the three side lengths. In a triangle the greatest angle is opposite the greatest side, so it is opposite $m + n$.

A is opposite $m + n$.

(a)(ii) The side opposite A is now known, so apply the cosine rule with the adjacent sides m and $m - n$.

$$(m + n)^2 = m^2 + (m - n)^2 - 2m(m - n)\cos A.$$

Rearranging gives

$$\cos A = \frac{m^2 + (m - n)^2 - (m + n)^2}{2m(m - n)} = \frac{m^2 - 4mn}{2m(m - n)} = \frac{m - 4n}{2(m - n)}.$$

$$\cos A = \frac{m - 4n}{2(m - n)}, \text{ as required.}$$

(b) An angle subtended by a diameter at the circumference is a right angle. Thus $A = 90^\circ$ and $\cos A = 0$. Use the expression just proved.

$$0 = \frac{m - 4n}{2(m - n)} \implies m - 4n = 0.$$

With $m = 8$,

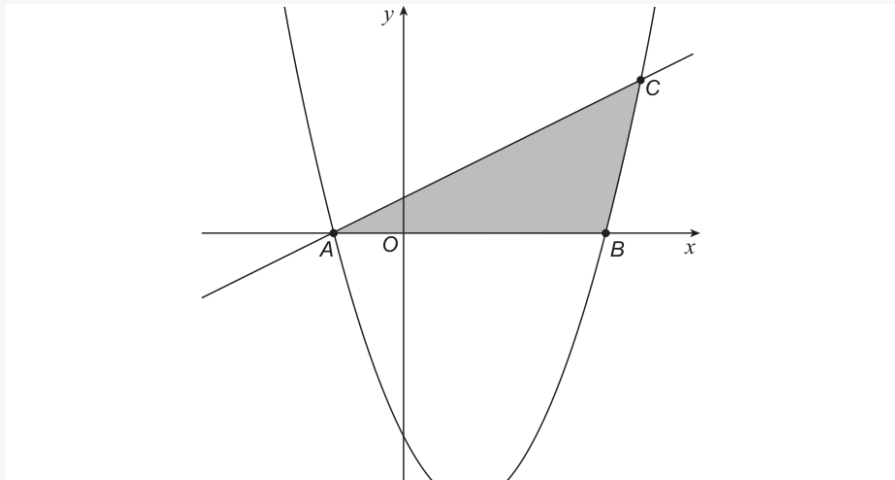
$$8 = 4n \implies n = 2.$$

$$\boxed{n = 2.}$$

Question 9

Question 9

The graphs are $y = x^2 - 4x - 12$ and $y = x + 2$.



- (a) Write three inequalities describing the shaded region. (2)
 (b) Find the coordinates of A, B, and C. (4)
 (c) Find the exact area of the shaded region, fully justifying your answer. (6)

Model solution

(a) Read each boundary from the diagram: the region is below the line, above the parabola, and above the x -axis.

$$y \leq x + 2, \quad y \geq x^2 - 4x - 12, \quad y \geq 0.$$

(b) Points A and B are the x -intercepts. Point C is where the line and parabola meet.

$$x + 2 = 0 \implies x = -2, \quad x^2 - 4x - 12 = 0 \implies (x - 6)(x + 2) = 0.$$

Thus $A = (-2, 0)$ and $B = (6, 0)$. For the other intersection,

$$x + 2 = x^2 - 4x - 12 \implies x^2 - 5x - 14 = 0 \implies (x + 2)(x - 7) = 0.$$

The non-A intersection has $x = 7$, and $y = 7 + 2 = 9$.

$$A = (-2, 0), \quad B = (6, 0), \quad C = (7, 9).$$

(c) Split the region at B. From A to C, the area below the straight line is a triangle of base 9 and height 9. The part below the x -axis and above the parabola from $x = 6$ to $x = 7$ must be removed.

$$\text{area below AC} = \frac{1}{2} \times 9 \times 9 = \frac{81}{2}.$$

$$\text{area to remove} = \int_6^7 (x^2 - 4x - 12) \, dx = \left[\frac{x^3}{3} - 2x^2 - 12x \right]_6^7 = \frac{13}{3}.$$

$$\text{shaded area} = \frac{81}{2} - \frac{13}{3} = \frac{217}{6}.$$

$$\frac{217}{6} \text{ square units.}$$

www.mathswithmatt.co.uk

Question 10

Question 10

Water temperature is modelled by $T = 20 - ae^{-kt}$, where T is in $^{\circ}\text{C}$ and t is minutes. Initially $T = 6$, and after 10 minutes $T = 12$.

- (a) Find a . (1)
- (b) Find k , to two significant figures. (3)
- (c) Estimate how long it takes to reach 18°C . (3)
- (d) Explain why the model may not be appropriate after three hours. (1)

Model solution

(a) At removal from the refrigerator $t = 0$, so the exponential factor is 1.

$$6 = 20 - ae^0 = 20 - a \implies a = 14.$$

$$a = 14.$$

(b) Use the second temperature reading. Isolating the exponential first makes logarithms the natural way to solve for k .

$$12 = 20 - 14e^{-10k} \implies e^{-10k} = \frac{4}{7}.$$

$$-10k = \ln\left(\frac{4}{7}\right) \implies k = -\frac{1}{10} \ln\left(\frac{4}{7}\right) = 0.05596\dots$$

$$k = 0.056 \text{ (two significant figures).}$$

(c) Set the model temperature to 18 and solve the exponential equation.

$$18 = 20 - 14e^{-0.056t} \implies e^{-0.056t} = \frac{1}{7}.$$

$$t = -\frac{\ln(1/7)}{0.056} = 34.7\dots$$

About 35 minutes.

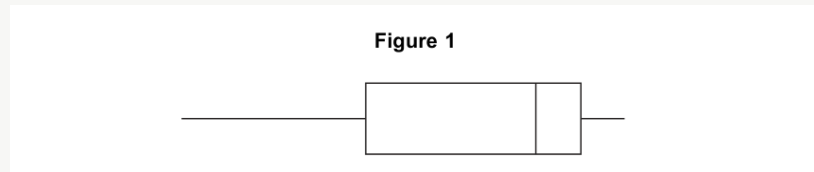
(d) The room temperature is assumed constant at 20°C ; over three hours this condition may change (and the water would in practice be effectively at room temperature).

The long-term prediction is unreliable because the modelling conditions may not remain true.

Question 11

Question 11

Choose the term that best describes the distribution represented by the boxplot.



(1)

Model solution

The longer tail is towards the lower values, so the distribution is skewed to the left.

Negatively skewed.

Question 12

Question 12

Shelly selects two of 15 groups at random and interviews every member of each selected group. State the sampling method. (1)

Model solution

The population is divided into natural groups, whole groups are selected at random, and everyone within them is sampled.

Cluster sampling.

www.mathswithmatt.co.uk

Question 13

Question 13

Two random samples of 12 NO_x emissions (g/km) were taken from the Large Data Set.

2002: 0.031, 0.019, 0.091, 0.025, 0.030, 0.061, 0.047, 0.029, 0.059, 0.363, 0.330, 0.376.

2016: 0.005, 0.047, 0.053, 0.063, 0.026, 0.013, 0.058, 0.012, 0.010, 0.010, 0.008, 0.008.

The 2002 mean and standard deviation are 0.122 and 0.137.

- (a) Find the 2016 mean and standard deviation, to three decimal places. (2)
- (b)(i) Show how the claim that the mean emissions fell by over 75% was obtained. (2)
- (b)(ii) Make two comments, using knowledge of the Large Data Set, on the validity of the claim that this applies to all makes of car in all UK areas. (2)

Model solution

(a) Use the sample values in the 2016 row to calculate the ordinary sample mean and standard deviation.

$$\bar{x} = 0.026 \dots, \quad s = 0.021 \dots$$

Mean = 0.026 g/km; standard deviation = 0.021 g/km.

(b)(i) Compare the fall in the two sample means with the original 2002 mean.

$$\frac{0.122 - 0.026}{0.122} \times 100 = 78.7 \dots \%$$

Since 78.7% > 75%, this is how Siti obtained "over 75%".

Estimated decrease = 78.7%, which is over 75%.

(b)(ii) The samples and the LDS have limited coverage, so they cannot support a claim about every make and every UK area.

The LDS contains only a limited number of car makes; it also does not cover cars from all areas of the UK. (The samples of 12 are also too small for such a broad conclusion.)

Question 14

Question 14

On each independent gym visit, $P(A) = 0.2$, $P(B) = 0.35$, $P(C) = 0.45$, for apple, banana, and cake respectively. For four visits find the probability of:

- (a) at least one banana; (2)
- (b) the same item each time; (2)
- (c) apple twice and cake twice. (3)

Model solution

(a) "At least one" is most efficiently found using its complement: no bananas on all four independent visits.

$$P(\text{at least one banana}) = 1 - P(\text{no bananas}) = 1 - (0.65)^4 = 0.82149375.$$

0.821 (approximately).

(b) The three mutually exclusive ways are four apples, four bananas, or four cakes; add their probabilities.

$$(0.2)^4 + (0.35)^4 + (0.45)^4 = 0.0576125.$$

0.0576 (approximately).

(c) Choose which two of the four visits are apples, then multiply by the probability of one such arrangement. There are $\binom{4}{2} = 6$ arrangements.

$$\binom{4}{2} (0.2)^2 (0.45)^2 = 6(0.2)^2 (0.45)^2 = 0.0486.$$

0.0486.

Question 15

Question 15

$$P(X = x) = \begin{cases} cx, & x = 1, 2, \\ kx^2, & x = 3, 4, \\ 0, & \text{otherwise,} \end{cases}$$

where k and c are constants.

(a) State $P(X = 3)$ in terms of k .

(1)

(b) Given $P(X \geq 3) = 3P(X \leq 2)$, find exact values of k and c .

(4)

Model solution

(a) For $x = 3$, use the kx^2 branch of the probability distribution.

$$P(X = 3) = k(3)^2 = 9k.$$

$$9k.$$

(b) First list the four probabilities. Their sum is 1. The given condition supplies a second equation.

$$P(X = 1) = c, \quad P(X = 2) = 2c, \quad P(X = 3) = 9k, \quad P(X = 4) = 16k.$$

$$3c + 25k = 1.$$

Also,

$$9k + 16k = 3(c + 2c) \implies 25k = 9c.$$

Thus $c = 25k/9$, so

$$3\left(\frac{25k}{9}\right) + 25k = 1 \implies \frac{100k}{3} = 1 \implies k = \frac{3}{100}.$$

$$c = \frac{25}{9} \cdot \frac{3}{100} = \frac{1}{12}.$$

$$k = \frac{3}{100}, \quad c = \frac{1}{12}.$$

Question 16

Question 16

A coin is believed to have $P(\text{head}) = 0.7$.

- (a)(i) Assuming this is true, find $P(\text{exactly 6 heads in 7 tosses})$. (1)
 (a)(ii) Find the mean number of heads in 7 tosses. (1)
 (b) Harry believes the probability of a head is greater than 0.7. In 35 tosses he obtains 28 heads. Carry out a hypothesis test at the 10% significance level. (5)

Model solution

(a)(i) Each toss is independent with constant head probability, so the number of heads has distribution $X \sim B(7, 0.7)$.

$$P(X = 6) = \binom{7}{6} (0.7)^6 (0.3) = 0.2470629 \dots$$

0.247 (approximately).

(a)(ii) For a binomial variable, the mean is np .

$$E(X) = 7(0.7) = 4.9.$$

4.9 heads.

(b) Harry's claim gives a one-tailed upper test. Under the null hypothesis the count of heads has a binomial distribution with $n = 35$ and $p = 0.7$.

$$H_0 : p = 0.7, \quad H_1 : p > 0.7, \quad Y \sim B(35, 0.7) \text{ under } H_0.$$

The observed result is $Y = 28$. The upper-tail p-value is

$$P(Y \geq 28) = 1 - P(Y \leq 27) = 1 - 0.86735 \dots = 0.13265 \dots$$

This is the probability, assuming the claimed bias is 0.7, of a result at least as favourable to Harry's claim as the one observed. Since $0.133 > 0.10$, it is not sufficiently unusual at the stated significance level.

Do not reject H_0 . There is insufficient evidence at the 10% level that the probability of a head is greater than 0.7.