



Edexcel AQA AS Mathematics

7356/2 - Pure Mathematics and Statistics

Thursday 25 May 2023

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 2	3
Question 3	4
Question 4	5
Question 5	6
Question 6	7
Question 7	8
Question 8	9
Question 9	10
Question 10	11
Question 11	13
Question 12	14
Question 13	15
Question 14	16
Question 15	17
Question 16	18
Question 17	19
Question 18	20
Question 19	21

Question 1

Question 1

Simplify $\log_2(8^a)$.

Model solution

Write 8 as a power of the base 2, then use the power rule for logarithms.

$$\log_2(8^a) = \log_2((2^3)^a) = \log_2(2^{3a}) = 3a.$$

$$3a$$

(1)

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Question 2

Question 2

Given $\sin y = \frac{4}{5}$ and $90^\circ < y < 180^\circ$, find $\cos y$.

Model solution

Use $\sin^2 y + \cos^2 y = 1$. The stated range puts y in quadrant II, where cosine is negative.

$$\cos y = -\sqrt{1 - \left(\frac{4}{5}\right)^2} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}.$$

$$\cos y = -\frac{3}{5}$$

(1)

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Question 3

Question 3

(a) Find $\int \left(2x^3 + \frac{8}{x^2} \right) dx$.

(b) A curve has gradient $\frac{dy}{dx} = 2x^3 + \frac{8}{x^2}$ and x-intercept $(2, 0)$. Find its equation.

Model solution

(a) Integrate each power separately, first writing $8/x^2$ as $8x^{-2}$. Include a constant because this is an indefinite integral.

$$\int (2x^3 + 8x^{-2}) dx = \frac{1}{2}x^4 - 8x^{-1} + c = \frac{1}{2}x^4 - \frac{8}{x} + c.$$

$$\frac{1}{2}x^4 - \frac{8}{x} + c$$

(3)

(b) The equation is the integral found in part (a). Use the given point to determine its constant.

$$y = \frac{1}{2}x^4 - \frac{8}{x} + c, \quad 0 = \frac{1}{2}(2)^4 - \frac{8}{2} + c = 8 - 4 + c,$$

$$c = -4.$$

$$y = \frac{1}{2}x^4 - \frac{8}{x} - 4$$

(2)

Question 4

Question 4

Find the exact solution of $\ln(x+1) + \ln(x-1) = \ln 15 - 2\ln 7$. Fully justify your answer.

Model solution

Combine logarithms before removing $\ln$. This turns the equation into one algebraic equation; the logarithm domains must then be checked.

$$\ln((x+1)(x-1)) = \ln 15 - \ln(7^2) = \ln\left(\frac{15}{49}\right).$$

$$x^2 - 1 = \frac{15}{49} \Rightarrow x^2 = \frac{64}{49} \Rightarrow x = \pm \frac{8}{7}.$$

Both $x+1$ and $x-1$ must be positive. For $x = -\frac{8}{7}$, they are negative, so the original logarithms do not exist.

$$x = \frac{8}{7}$$

(5)

Question 5

Question 5

Given $\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$ and $\cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$, show that $\tan 15^\circ = 2 - \sqrt{3}$.

Model solution

Use $\tan \theta = \sin \theta / \cos \theta$. Rationalising the denominator is appropriate because it removes the surd denominator:

$$\tan 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{(\sqrt{6}-\sqrt{2})^2}{6-2}.$$

$$\frac{6-2\sqrt{12}+2}{4} = \frac{8-4\sqrt{3}}{4} = 2-\sqrt{3}.$$

$$\tan 15^\circ = 2 - \sqrt{3}$$

(4)

Question 6

Question 6

The curve $y = 2x^2 + px + 1$ and the line $y = 5x - 2$ intersect. Find the set of values of p for which they meet at two distinct points.

Model solution

Intersection points give a quadratic in x . Two distinct intersections mean two distinct real roots, so its discriminant must be positive.

$$2x^2 + px + 1 = 5x - 2 \Rightarrow 2x^2 + (p - 5)x + 3 = 0.$$

$$\Delta = (p - 5)^2 - 4(2)(3) = (p - 5)^2 - 24 > 0.$$

$$(p - 5)^2 > 24 = (2\sqrt{6})^2 \Rightarrow p - 5 > 2\sqrt{6} \text{ or } p - 5 < -2\sqrt{6}.$$

$$p > 5 + 2\sqrt{6} \text{ or } p < 5 - 2\sqrt{6}$$

(5)

Question 7

Question 7

Curve C : $y = f(x)$ has maximum $(a, 2b)$. It is transformed to (a) $y = f(x + 2)$, (b) $y = 4f(x)$, and (c) $y = f(3x)$. State the requested maximum coordinates or scale factor.

Model solution

(a) Replacing x by $x + 2$ translates every point 2 units left.

Maximum $(a - 2, 2b)$

(1)

(b) Multiplying $f(x)$ by 4 is a vertical stretch, so only the y -coordinate changes.

Maximum $(a, 8b)$

(1)

(c) In $f(3x)$, x -coordinates are divided by 3, so this is a stretch parallel to the x -axis with scale factor $\frac{1}{3}$.

Scalefactor $\frac{1}{3}$

(1)

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Question 8

Question 8

Prove that the sum of the cubes of two consecutive odd numbers is always a multiple of 4.

Model solution

Represent consecutive odd integers symmetrically as $2k - 1$ and $2k + 1$, where k is an integer. Expand their cubes and factor the sum.

$$(2k + 1)^3 = 8k^3 + 12k^2 + 6k + 1,$$

$$(2k - 1)^3 = 8k^3 - 12k^2 + 6k - 1.$$

$$(2k + 1)^3 + (2k - 1)^3 = 16k^3 + 12k = 4k(4k^2 + 3).$$

Since k and $4k^2 + 3$ are integers, this has a factor of 4. Therefore the sum is always a multiple of 4.

Proved.

(5)

Question 9

Question 9

Selling price P is inversely proportional to number produced n , with $P = 24$ when $n = 10$. Production cost C is inversely proportional to n^2 , with $C = 60$ when $n = 10$. (a) Show $P = 240/n$. (b) Find n for which $P > C$. (c) Explain its significance.

Model solution

(a) Inverse proportionality has the form $P = k/n$. Use the given pair of values to find k .

$$24 = \frac{k}{10} \Rightarrow k = 240.$$

$$P = \frac{240}{n}$$

(1)

(b) The phrase "inversely proportional to the square" gives $C = l/n^2$. Compare the two quantities after finding l .

$$60 = \frac{l}{10^2} \Rightarrow l = 6000, \quad C = \frac{6000}{n^2}.$$

$$\frac{240}{n} > \frac{6000}{n^2}.$$

As n is a positive number of items, multiplying by n^2 preserves the inequality.

$$240n > 6000 \Rightarrow n > 25.$$

$$n > 25$$

(4)

(c) Here price exceeds production cost exactly when more than 25 items are produced and sold.

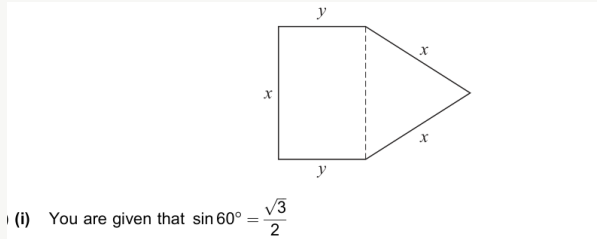
The artist makes a profit if more than 25 items are sold.

(1)

Question 10

Question 10

A 66 cm wire makes three sides of a rectangle of sides x, y , and two sides of an equilateral triangle of side x . (a)(i) Explain the triangle area $\frac{\sqrt{3}}{4}x^2$. (a)(ii) Show $A = 33x - \frac{1}{4}(6 - \sqrt{3})x^2$. (b) Find the x giving maximum area.



Model solution

(a)(i) The two sides around the included 60° angle both have length x , so use $\frac{1}{2}ab \sin C$.

$$\text{Area} = \frac{1}{2}x \cdot x \sin 60^\circ = \frac{1}{2}x^2 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}x^2.$$

$$\frac{\sqrt{3}}{4}x^2 \text{ cm}^2$$

(1)

(a)(ii) The outside wire comprises three lengths x and two lengths y , giving the constraint needed to eliminate y . The enclosed area is rectangle plus triangle.

$$3x + 2y = 66 \Rightarrow y = 33 - \frac{3}{2}x.$$

$$A = xy + \frac{\sqrt{3}}{4}x^2 = x \left(33 - \frac{3}{2}x \right) + \frac{\sqrt{3}}{4}x^2 = 33x - \frac{1}{4}(6 - \sqrt{3})x^2.$$

$$A = 33x - \frac{1}{4}(6 - \sqrt{3})x^2$$

(3)

(b) Differentiate the area function and set its derivative to zero to locate a stationary area. The second derivative establishes whether it is a maximum.

$$\frac{dA}{dx} = 33 - \frac{1}{2}(6 - \sqrt{3})x.$$

$$0 = 33 - \frac{1}{2}(6 - \sqrt{3})x \Rightarrow (6 - \sqrt{3})x = 66 \Rightarrow x = \frac{66}{6 - \sqrt{3}}.$$

$$x = \frac{66(6 + \sqrt{3})}{36 - 3} = 12 + 2\sqrt{3}.$$

$$\frac{d^2A}{dx^2} = -\frac{1}{2}(6 - \sqrt{3}) < 0,$$

so the stationary value gives a maximum area.

$$x = 12 + 2\sqrt{3} \text{ cm}$$

(7)

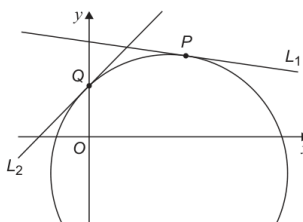
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Question 11

Question 11

$L_1 : x + 7y - 41 = 0$ is tangent to circle C at $P(6, 5)$, and $L_2 : y = x + 3$ is tangent at $Q(0, 3)$. (a) Show the radius through P is $y = 7x - 37$. (b) Find C .

The lines L_1 and L_2 and the circle C are shown in the diagram below.



Model solution

(a) A radius to a tangent point is perpendicular to the tangent. First find the tangent gradient, then use the negative reciprocal and point--gradient form through P .

$$L_1 : y = -\frac{1}{7}x + \frac{41}{7}, \quad m_{\text{radius}} = 7.$$

$$y - 5 = 7(x - 6) \Rightarrow y = 7x - 37.$$

$$y = 7x - 37$$

(3)

(b) The centre lies on both radii. Since L_2 has gradient 1, its radius through Q has gradient -1 .

$$y - 3 = -(x - 0) \Rightarrow y = -x + 3.$$

$$7x - 37 = -x + 3 \Rightarrow x = 5, \quad y = -2.$$

The centre is $(5, -2)$. Its squared radius, using Q , is $5^2 + (-5)^2 = 50$.

$$(x - 5)^2 + (y + 2)^2 = 50$$

(4)

Question 12

Question 12

Identify the continuous variable: flavour, known standard deviation of mass, mass of an individual bag, or number of bags in a box.

Model solution

A continuous variable can take any value within an interval, including decimal values. Mass is measured on a continuous scale.

The mass of an individual bag of nuts.

(1)

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Question 13

Question 13

For continuous variable X , the frequencies are

x	$11 < x \leq 21$	$21 < x \leq 24$	$24 < x \leq 42$	$42 < x \leq 50$
Frequency	7	9	36	18

Find the frequency density for $24 < x \leq 42$.

Model solution

Histogram height is frequency density, so divide the frequency by the class width. This class has width $42 - 24 = 18$.

$$\text{frequency density} = \frac{36}{18} = 2.$$

2

(1)

Question 14

Question 14

A factory has 65 production and 28 office employees. (a) Explain how to take a simple random sample of 20 from all 93. (b) State a different representative sampling method.

Model solution

(a) A simple random sample gives every employee an equal chance. Number the full population, then use random numbers rather than selecting conveniently.

Label employees 1 to 93; generate random integers from 1 to 93.

Select the employees with those labels, ignoring any repeated number, until 20 different employees have been selected.

Number 1--93; randomly select 20 distinct labels.

(3)

(b) To make both employee groups represented in their population proportions, divide the population into production and office strata before sampling.

Stratified sampling.

(1)

Question 15

Question 15

Bowl A contains 1, 2, 3, 7; B contains 0, 0, 2, 3, 5, 6, 8, 9; C contains 0, 1, 1, 2, 3, 3, 3, 6, 7. One ball is drawn from each to form a three-digit number. Find the probability that it is (a) even, (b) 703, (c) divisible by 111.

In bowl C there are nine balls numbered 0, 1, 1, 2, 3, 3, 3, 6 and 7.

This information is shown in the diagram below.

A three-digit number is generated using the following method:

- a ball is selected at random from each bowl
- the first digit of the number is the ball drawn from bowl A
- the second digit of the number is the ball drawn from bowl B
- the third digit of the number is the ball drawn from bowl C

5 (a) Find the probability that the number generated is even.

Model solution

(a) A number is even exactly when its final digit is even. Bowl C has three even-numbered balls (0, 2 and 6) out of nine.

$$\frac{3}{9} = \frac{1}{3}$$

(1)

(b) The draws are from different bowls, so multiply their independent selection probabilities. There are two 0s in B and three 3s in C.

$$P(703) = \frac{1}{4} \times \frac{2}{8} \times \frac{3}{9} = \frac{1}{48}.$$

$$\frac{1}{48}$$

(2)

(c) The possible three-digit multiples of 111 that can be made are 222 and 333. Add their mutually exclusive probabilities.

$$P(222) + P(333) = \frac{1}{4} \cdot \frac{1}{8} \cdot \frac{1}{9} + \frac{1}{4} \cdot \frac{1}{8} \cdot \frac{3}{9} = \frac{1}{72}.$$

$$\frac{1}{72}$$

(2)

Question 16

Question 16

x	1	2	3	4	5	6
$P(X = x)$	0.3	0.1	0.2	0.1	0.1	0.2

y	2	3	4	5	6	7
$P(Y = y)$	0.3	p	0.2	0.1	p	$3p + 0.05$

Determine whether the claim $P(X \geq 3) > P(Y \leq 4)$ is correct.

Model solution

First determine p , because probabilities in a distribution must sum to 1. Then calculate the two probabilities in the claim.

$$0.3 + p + 0.2 + 0.1 + p + (3p + 0.05) = 1,$$

$$0.65 + 5p = 1 \Rightarrow p = 0.07.$$

$$P(X \geq 3) = 0.2 + 0.1 + 0.1 + 0.2 = 0.6,$$

$$P(Y \leq 4) = 0.3 + 0.07 + 0.2 = 0.57.$$

The claim is correct, since $0.6 > 0.57$.

(4)

Question 17

Question 17

An archer has 30 attempts per session and hits the centre with probability 0.79 each time. The number of hits is binomial. Find (a) its mean, (b) $P(X = 22)$, (c) $P(X \leq 18)$, and (d) $P(X > 26)$.

Model solution

Let $X \sim \text{Bin}(30, 0.79)$, since there is a fixed number of attempts, each has two outcomes and a constant hit probability. (a) For a binomial distribution, $E(X) = np$.

$$E(X) = 30(0.79) = 23.7.$$

23.7

(1)

(b) Evaluate the binomial probability at exactly 22 hits.

$$P(X = 22) = \binom{30}{22} (0.79)^{22} (0.21)^8 = 0.124 \text{ (3 s.f.)}$$

0.124

(1)

(c) Use the cumulative binomial probability for "18 or fewer".

$$P(X \leq 18) = 0.01399 \dots$$

0.014(3s.f.)

(1)

(d) "More than 26" means $X \geq 27$. Use the complement of the easier cumulative probability.

$$P(X > 26) = 1 - P(X \leq 26) = 1 - 0.9015596 \dots = 0.0984404 \dots$$

0.0984(3s.f.)

(2)

Question 18

Question 18

It is believed 25% of bakery customers buy a loaf. After free samples, a large random sample is tested at 5%; the test statistic lies in the critical region. (a) State hypotheses. (b) Give the contextual conclusion.

Model solution

(a) The possible effect being investigated is an increase, so use a one-tailed test. Let p be the population proportion of customers buying a loaf after samples are provided.

$$H_0 : p = 0.25, H_1 : p > 0.25$$

(1)

(b) Being in the critical region at the 5% level means the result is sufficiently unlikely under H_0 , so reject H_0 .

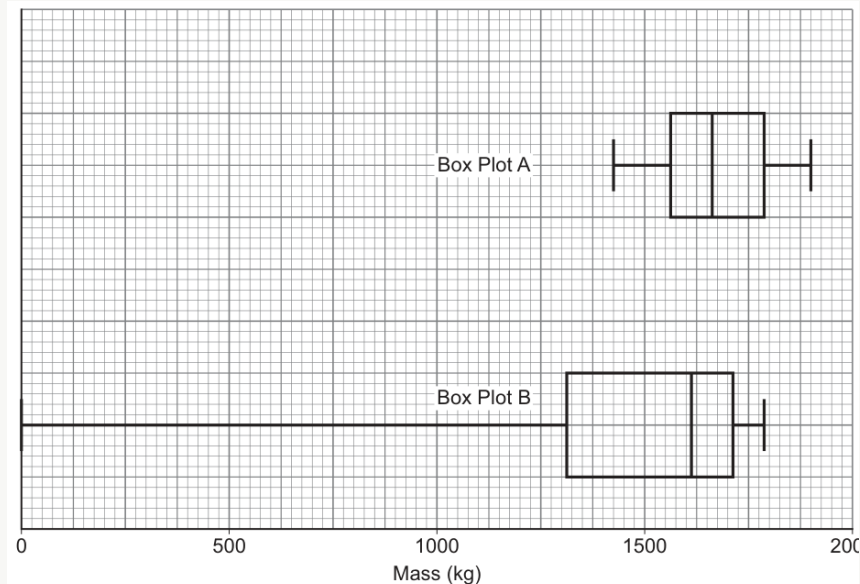
There is sufficient evidence to suggest that the proportion of customers buying a loaf of bread has increased.

(2)

Question 19

Question 19

Box plots A and B compare masses of samples of 20 convertible cars from the Large Data Set in 2002 and 2016. (a) Estimate median of A. (b)(i) Why might B be claimed incorrect? (ii) Comment. (c) Explain why B must be 2002 data.



19 (a) Estimate the median of the masses from Box Plot A

[1 mark]

Model solution

(a) Read the median line inside Box Plot A against the mass scale.

Approximately 1662.5 kg (acceptable estimate 1650 to 1675 kg).

(1)

(b)(i) Box Plot B has its minimum at 0 kg, apparently implying a car has zero mass.

It implies at least one car has mass 0 kg, which appears impossible.

(1)

(b)(ii) The plot need not be wrong: the Large Data Set contains zero entries in the car-mass field.

The claim is invalid; zero masses do occur in the Large Data Set.

(1)

(c) Of the two relevant years, only the 2002 data contain zero entries in the masses field. The zero minimum on B therefore identifies its year.

Box Plot B is from 2002 because only the 2002 data contain zero masses.

(1)

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