



Edexcel AQA AS Mathematics

7356/2 - Pure Mathematics and Statistics

Thursday 23 May 2024

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2024 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 2	3
Question 3	4
Question 4	5
Question 5	6
Question 6	7
Question 7	8
Question 8	9
Question 9	10
Question 10	11
Question 11	13
Question 12	14
Question 13	15
Question 14	16
Question 15	17
Question 16	18
Question 17	19

Question 1

Question 1

Line L has equation $5y = 4x + 6$. Find the gradient of a line parallel to L.

(1)

Model solution

Put the line into $y = mx + c$ form: the coefficient of x is its gradient. Parallel lines have equal gradients.

$$5y = 4x + 6 \Rightarrow y = \frac{4}{5}x + \frac{6}{5}.$$

$$\frac{4}{5}$$

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Question 2

Question 2

Identify the equation that is true for all x .

(1)

Model solution

Use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ and rearrange it to make $\cos^2 x$ the subject.

$$\cos^2 x = 1 - \sin^2 x.$$

$$\cos^2 x = 1 - \sin^2 x$$

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Question 3

Question 3

Given $3 \log_a x = \log_a 72 - 2 \log_a 3$, solve for x , fully justifying the answer. (4)

Model solution

The coefficients of logarithms suggest the power law. Combining both sides into single logarithms lets us use the one-to-one property of $\log_a$.

$$3 \log_a x = \log_a (x^3), \quad 2 \log_a 3 = \log_a (3^2) = \log_a 9,$$

$$\log_a (x^3) = \log_a 72 - \log_a 9 = \log_a \left(\frac{72}{9} \right) = \log_a 8.$$

Since logarithms to the same valid base are equal only when their arguments are equal, $x^3 = 8$. The positive solution also satisfies the domain condition $x > 0$.

$$x = 2.$$

$$x = 2$$

Question 4

Question 4

Curve C has equation $y = 8 \sin x$.

- (a) It is translated by vector $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ to give C_1 . Find C_1 . (1)
- (b) It is stretched by scale factor 4 in the y direction to give C_2 . Find C_2 . (1)
- (c) It is stretched by scale factor 2 in the x direction to give C_3 . Find C_3 . (1)

Model solution

(a) A vertical translation adds 4 to every y -value.

$$y = 8 \sin x + 4.$$

$$C_1 : y = 8 \sin x + 4$$

(b) A stretch in the y direction multiplies every output by 4.

$$y = 4(8 \sin x) = 32 \sin x.$$

$$C_2 : y = 32 \sin x$$

(c) A horizontal stretch by factor 2 makes the graph twice as wide, so replace x by $x/2$.

$$y = 8 \sin \left(\frac{x}{2} \right).$$

$$C_3 : y = 8 \sin \left(\frac{x}{2} \right)$$

Question 5

Question 5

A student claims that $4n^2 + 3$ is prime for every positive integer n . Disprove the claim using a counterexample, fully justifying it. (3)

Model solution

One positive integer producing a composite value is enough to disprove a claim about every positive integer. Try small values of n .

$$n = 3: \quad 4n^2 + 3 = 4(3)^2 + 3 = 39 = 3 \times 13.$$

Both factors are integers greater than 1, so 39 is not prime.

For $n = 3$, the expression is 39, which is not prime; therefore the statement is false.

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Question 6

Question 6

In $(3 + ax)^n$, where a and n are integers, the coefficient of x^2 is 4860.

- (a) Show that $3^n a^2 n(n-1) = 87480$. (3)
 (b) The constant term is 729 and the coefficient of x is negative. (i) Verify $n = 6$. (1)
 (b) (ii) Find a . (3)

Model solution

(a) In the binomial expansion, an x^2 term is formed by choosing $(ax)^2$ and choosing 3 from the other $n-2$ factors.

$$\binom{n}{2} 3^{n-2} (ax)^2 = 4860x^2.$$

Equating coefficients and writing $\binom{n}{2} = \frac{n(n-1)}{2}$ gives

$$\frac{n(n-1)}{2} 3^{n-2} a^2 = 4860 \Rightarrow 3^n a^2 n(n-1) = 2 \times 9 \times 4860 = 87480.$$

$$3^n a^2 n(n-1) = 87480$$

(b)(i) The constant term comes from choosing 3 from every factor, so it is 3^n .

$$3^n = 729 = 3^6 \Rightarrow n = 6.$$

$$n = 6$$

(b)(ii) Substitute this value into the equation proved in part (a), then use the sign of the x coefficient to choose between the two square roots.

$$3^6 a^2 (6)(5) = 87480 \Rightarrow 729 a^2 \times 30 = 87480 \Rightarrow a^2 = 4.$$

$$a = \pm 2.$$

The coefficient of x is $\binom{6}{1} 3^5 a$, whose sign is the sign of a ; it is negative, so $a < 0$.

$$a = -2$$

Question 7

Question 7

$A = (4, 1)$ and $B = (-8, 5)$.

- (a) Find the equation of the perpendicular bisector of AB. (5)
 (b) A circle passes through A and B, and a diameter lies along the x-axis. Find its equation. (4)

Model solution

(a) A perpendicular bisector passes through the midpoint of the segment and has gradient the negative reciprocal of the segment's gradient.

$$M = \left(\frac{4 + (-8)}{2}, \frac{1 + 5}{2} \right) = (-2, 3), \quad m_{AB} = \frac{5 - 1}{-8 - 4} = -\frac{1}{3}.$$

$$m_{\perp} = 3, \quad y - 3 = 3(x + 2), \quad y = 3x + 9.$$

$$y = 3x + 9$$

(b) The centre lies on the perpendicular bisector and on the x-axis. Their intersection gives the centre, then the distance to A gives the radius.

$$0 = 3x + 9 \Rightarrow x = -3, \quad \text{centre } (-3, 0),$$

$$r^2 = (4 - (-3))^2 + (1 - 0)^2 = 7^2 + 1^2 = 50.$$

$$(x + 3)^2 + y^2 = 50$$

Question 8

Question 8

Prove that the curve $y = x^3 + 15x - \frac{18}{x}$ has no stationary points.

(5)

Model solution

Stationary points require $\frac{dy}{dx} = 0$. Rewrite the reciprocal term as a negative power before differentiating.

$$y = x^3 + 15x - 18x^{-1}, \quad \frac{dy}{dx} = 3x^2 + 15 + 18x^{-2} = 3x^2 + 15 + \frac{18}{x^2}.$$

For a stationary point, with $x \neq 0$, multiply by x^2 to clear the denominator:

$$3x^2 + 15 + \frac{18}{x^2} = 0 \Rightarrow 3x^4 + 15x^2 + 18 = 0 \Rightarrow x^4 + 5x^2 + 6 = 0,$$

$$(x^2 + 2)(x^2 + 3) = 0.$$

This would require $x^2 = -2$ or $x^2 = -3$, neither of which has a real solution. Therefore the derivative is never zero for a real point on the curve.

The curve has no stationary points.

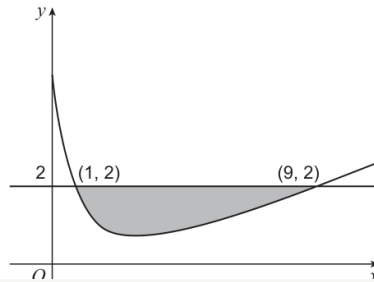
Question 9

Question 9

The curve $y = x - a\sqrt{x} + b$ meets $y = 2$ at $(1, 2)$ and $(9, 2)$.

where a and b are constants.

The curve intersects the line $y = 2$ at points with coordinates $(1, 2)$ and $(9, 2)$, as shown in the diagram below.



(a) Show that $a = 4$ and find b .

(3)

(b) Show that the enclosed area is $\frac{16}{3}$.

(6)

Model solution

(a) Each given intersection lies on both the curve and $y = 2$, so substitution supplies two simultaneous equations.

$$2 = 1 - a + b \Rightarrow 1 = -a + b, \quad 2 = 9 - 3a + b \Rightarrow -7 = -3a + b.$$

$$(-7) - 1 = (-3a + b) - (-a + b) \Rightarrow -8 = -2a \Rightarrow a = 4,$$

$$1 = -4 + b \Rightarrow b = 5.$$

$$a = 4, \quad b = 5$$

(b) Between $x = 1$ and $x = 9$, the line is above the curve, so calculate the rectangle under $y = 2$ and subtract the area under the curve.

$$\int_1^9 (x - 4\sqrt{x} + 5) \, dx = \left[\frac{x^2}{2} - \frac{8}{3}x^{3/2} + 5x \right]_1^9 = \frac{32}{3}.$$

$$\text{Area under } y = 2 = 2(9 - 1) = 16, \quad \text{enclosed area} = 16 - \frac{32}{3} = \frac{16}{3}.$$

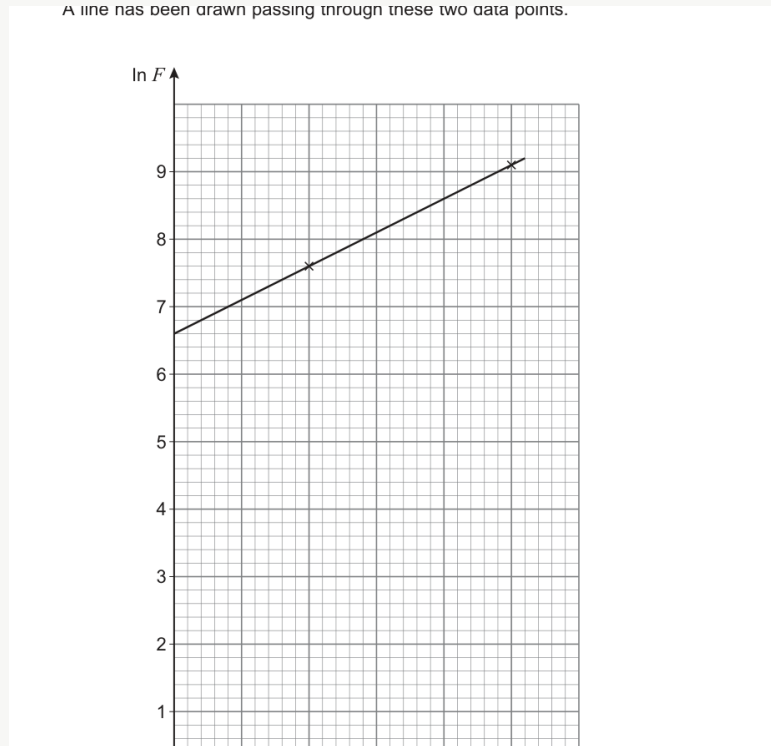
$$\frac{16}{3} \text{ square units}$$

Question 10

Question 10

Followers are modelled by $F = ae^{kt}$, where t is days after release. $F = 2050$ when $t = 2$ and $F = 9200$ when $t = 5$.

A line has been drawn passing through these two data points.



- (a)(i) Show that $\ln F = \ln a + kt$. (2)
 (a)(ii) Use the graph to estimate a and k . (4)
 (b)(i) Show that $\frac{dF}{dt} = kF$. (2)
 (b)(ii) Estimate the rate at which followers are increasing when $t = 5$. (2)
 (c) Comment on the claim that after 30 days there will be more than one billion followers. (1)

Model solution

(a)(i) Taking natural logarithms is appropriate because it turns the exponential model into a straight-line form for the plotted graph.

$$\ln F = \ln(ae^{kt}) = \ln a + \ln(e^{kt}) = \ln a + kt.$$

$$\ln F = \ln a + kt$$

(a)(ii) Compare $\ln F = \ln a + kt$ with $y = c + mx$. The intercept is $\ln a$ and the gradient is k . From the drawn line, $\ln a \approx 6.6$ and $k \approx 0.5$.

$$a \approx e^{6.6} \approx 735, \quad k \approx 0.50.$$

$$a \approx 735, \quad k \approx 0.50$$

(b)(i) Differentiate the exponential model. The chain rule supplies the factor k , then replace ae^{kt} by F .

$$\frac{dF}{dt} = a \cdot ke^{kt} = k(ae^{kt}) = kF.$$

$$\frac{dF}{dt} = kF$$

(b)(ii) At $t = 5$, the model gives $F = 9200$. Substitute this and the estimated growth constant into the rate equation.

$$\frac{dF}{dt} \approx 0.50(9200) = 4600.$$

approximately 4600 followers per day

(c) This extrapolates a model fitted using only days 2 and 5 far beyond the observed data. Exponential growth is unlikely to continue at this rate as interest in the song will not keep increasing indefinitely.

The claim is unlikely to be reliable; the model is not likely to remain valid for 30 days

Question 11

Question 11

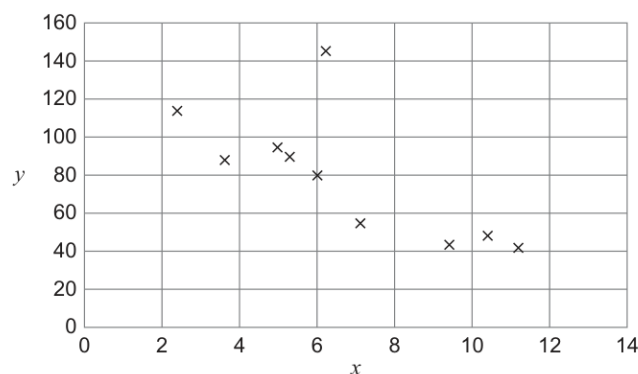
The table and scatter diagram give daily salt intake x (g) and Vitamin C intake y (mg) for adults A–J. Identify the outlier from A, B, E and J.

Answer **all** questions in the spaces provided.

The table below shows the daily salt intake, x grams, and the daily Vitamin C intake, y milligrams, for a group of 10 adults.

Adult	A	B	C	D	E	F	G	H	I	J
x	5.3	6.2	3.6	10.4	2.4	9.4	6	5	7.1	11.2
y	90	145	88	48	114	44	80	95	55	41

A scatter diagram of the data is shown below.



One of the adults is an outlier. Identify the letter of the adult that is the outlier.

Circle your answer below.

[1 mark]

A

B

E

J

(1)

Model solution

An outlier is a point noticeably separated from the overall negative association. Adult B has high Vitamin C intake for a moderate salt intake, unlike the general pattern.

B

Question 12

Question 12

Which is not a measure of spread: median, range, standard deviation or variance?

(1)

Model solution

Range, standard deviation and variance describe how dispersed data are. The median instead describes the central position.

median

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Question 13

Question 13

A stratified random sample of 50 is selected by year group from a school of 720 students.

Year group	10	11	12	13
Number of students	200	240	150	130

- (a) Find the number selected from each year group. (3)
 (b) State one advantage of a stratified random sample. (1)

Model solution

(a) Stratification keeps each group's proportion in the sample the same as in the school, so multiply each group size by $50/720$.

Year group	10	11	12	13
$\frac{\text{group size}}{720} \times 50$	13.9	16.7	10.4	9.0

Rounding to whole students while retaining a total of 50 gives

Year10 : 14; Year11 : 17; Year12 : 10; Year13 : 9.

(b) Each year group is represented in the sample in the correct proportion, making the sample more representative of the school.

It ensures every year group is represented proportionately.

Question 14

Question 14

$X \sim B(40, p)$ and $Y \sim B(25, 0.6)$. The mean of X equals the variance of Y .

(a) Find p .

(3)

(b) Find $P(Y = 17)$.

(1)

Model solution

(a) For $B(n, p)$, the mean is np and the variance is $np(1 - p)$. Use the stated equality.

$$E(X) = 40p, \quad \text{Var}(Y) = 25(0.6)(0.4) = 6,$$

$$40p = 6 \Rightarrow p = \frac{6}{40} = \frac{3}{20}.$$

$$p = \frac{3}{20}$$

(b) Use the binomial probability formula with $n = 25$, $p = 0.6$ and $r = 17$.

$$P(Y = 17) = \binom{25}{17} (0.6)^{17} (0.4)^8 = 0.119979 \dots$$

$$P(Y = 17) \approx 0.120$$

Question 15

Question 15

For the number of flowers X on a plant:

x	0	1	2	3	4	5 or more
$P(X = x)$	0.03	0.15	0.22	0.31	0.09	p

- (a) Find p . (2)
 (b) Two plants are randomly selected from a large batch. Find the probability that they produce a total of three flowers. (3)
 (c)(i) State one assumption needed for part (b). (1)
 (c)(ii) Comment on whether this assumption is likely to be valid in reality. (1)

Model solution

(a) The probabilities in a complete distribution must add to 1.

$$0.03 + 0.15 + 0.22 + 0.31 + 0.09 + p = 1 \Rightarrow p = 0.20.$$

$$p = 0.20$$

(b) A total of three can occur as (0,3), (1,2), (2,1) or (3,0). For independent plants, multiply within each pair and add the mutually exclusive cases.

$$P(\text{total } 3) = 0.03(0.31) + 0.15(0.22) + 0.22(0.15) + 0.31(0.03)$$

$$= 2(0.0093) + 2(0.033) = 0.0846.$$

$$0.0846$$

(c)(i) The multiplication used in part (b) requires the flower counts on the two selected plants to be independent.

The number of flowers on one selected plant is independent of the number on the other.

(c)(ii) This may not be fully realistic: plants from the same batch may have been grown in similar conditions, so their flower counts could be related.

The independence assumption may be questionable because common growing conditions can make the outcomes dependent.

Question 16

Question 16

For hydrocarbon emissions X (g/km) in the Large Data Set,

$$\sum x = 128.657, \quad \sum x^2 = 8.701\,707, \quad n = 2405.$$

- (a)(i) Find the mean of X . (1)
 (a)(ii) Find the standard deviation of X . (2)
 (b)(i) State, with a reason, whether these should be reliable estimates for the entire UK Department for Transport Stock Vehicle Database. (1)
 (b)(ii) State one emission type for which more than 80% of the entire database is known. (1)

Model solution

(a)(i) The mean is the total of the observations divided by the number of observations.

$$\bar{x} = \frac{128.657}{2405} = 0.053496 \dots$$

$$\bar{x} \approx 0.0535 \text{ g/km}$$

(a)(ii) Use the standard-deviation formula for data given as $\sum x$ and $\sum x^2$.

$$s = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{8.701707}{2405} - \left(\frac{128.657}{2405}\right)^2} = 0.0275 \dots$$

$$\text{standard deviation} \approx 0.0275 \text{ g/km}$$

(b)(i) The claim may be incorrect because the Large Data Set is not a representative sample of every car in the UK database: for example, it does not include every UK region.

Not necessarily; the LDS omits some regions/cars, so it may not represent the whole database.

(b)(ii) One accepted emission type with over 80% coverage is carbon dioxide.



Question 17

Question 17

The city proportion of vegans is thought to be 8%. An organic café owner believes the proportion of their customers who are vegan is greater than 8%. In a random sample of 50 café customers, 7 were vegan. Investigate this claim at the 5% level. (5)

Model solution

Let p be the proportion of the café's customers who are vegan. This is a one-tailed test because the owner's claim is specifically that the proportion is greater than 8%.

$$H_0 : p = 0.08, \quad H_1 : p > 0.08.$$

Under H_0 , each of the 50 independently sampled customers has probability 0.08 of being vegan, so the number X of vegan customers has a binomial distribution. The observed value is 7; values at least this large support H_1 .

$$X \sim B(50, 0.08), \quad P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.898128 \dots = 0.101872 \dots$$

The p -value is greater than 0.05, so the result is not significant at the 5% level and H_0 is not rejected.

There is insufficient evidence at the 5% level to conclude that the proportion of the