Maths with Matt

Physics and maths resources

OCR A Level Mathematics B (MEI)

H640/01 - Pure Mathematics and Mechanics

Tuesday 6 June 2023

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Tuesday 6 June 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A ball is thrown vertically upwards with a speed of 8 m s^{-1}. Find the times at which the ball is 3 m above the point of projection.

Model solution

Take upwards as positive, so $u = 8$, $a = -9.8$, *and the displacement is* $s = 3$.

$$3 = 8t - 4.9t^2 \quad \Rightarrow \quad 49t^2 - 80t + 30 = 0.$$

$$t = \frac{80 \pm \sqrt{80^2 - 4(49)(30)}}{98} = \frac{40 \pm \sqrt{130}}{49}.$$

$t = \dfrac{40 - \sqrt{130}}{49}$ ***s or*** $\dfrac{40 + \sqrt{130}}{49}$ ***s.*** ***(2)***

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Question 2

Question 2

Express $\dfrac{5x+1}{x^2-x-12}$ in partial fractions.

Model solution

Factor the denominator first, then use one constant numerator for each distinct linear factor.

$$x^2-x-12=(x-4)(x+3), \qquad \frac{5x+1}{(x-4)(x+3)}=\frac{A}{x-4}+\frac{B}{x+3}.$$

$$5x+1=A(x+3)+B(x-4).$$

Putting $x=4$ isolates A, and putting $x=-3$ isolates B:

$$21=7A\Rightarrow A=3, \qquad -14=-7B\Rightarrow B=2.$$

$$\frac{5x+1}{x^2-x-12}=\frac{3}{x-4}+\frac{2}{x+3}. \qquad \textit{(4)}$$

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Question 3

Question 3

Find $\int (2x^4 - x\sqrt{x})\,\mathrm{d}x$.

Model solution

Write the root as a fractional power, so that the power rule can be applied term by term.

$$\int (2x^4 - x^{3/2})\,\mathrm{d}x = \frac{2x^5}{5} - \frac{x^{5/2}}{5/2} + c.$$

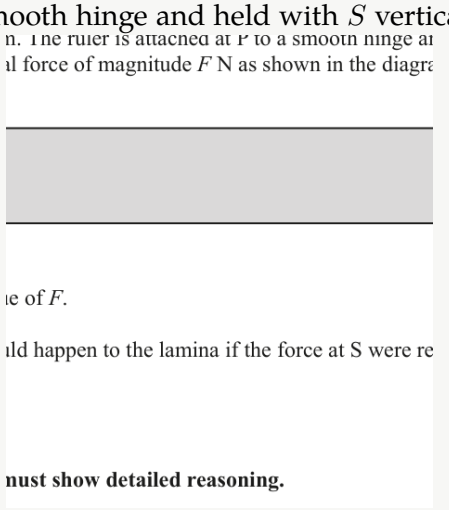
$$\frac{2x^5}{5} - \frac{2x^{5/2}}{5} + c. \qquad \textit{(3)}$$

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Question 4

Question 4

A ruler $PQRS$ is a uniform rectangular lamina with mass 20 grams. The length of PQ is 30 cm and the length of PS is 4 cm. The ruler is attached at P to a smooth hinge and held with S vertically below P by a horizontal force of magnitude F N as shown.



(a) Calculate F. (b) Explain what would happen to the lamina if the force at S were removed.

Model solution

(a) A uniform lamina has its weight acting at its centre, 15 cm horizontally from the hinge. Taking moments about P removes the unknown hinge reaction.

$$F(0.04) = (0.020g)(0.15).$$

$$F = \frac{0.020 \times 9.8 \times 0.15}{0.04} = 0.735.$$

$F = 0.735$ **N**. *(3)*

(b) With this balancing force absent, the weight gives a clockwise moment about P.

The lamina rotates clockwise about P. *(1)*

Question 5

Question 5

In this question you must show detailed reasoning. (a) Find the coordinates of the two stationary points on $y = 15 - x^2 - \dfrac{16}{x^2}$. (b) Show that both these stationary points are maximum points.

Model solution

(a) Stationary points occur where the first derivative is zero.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -2x + 32x^{-3} = 0 \Rightarrow -2x^4 + 32 = 0 \Rightarrow x^4 = 16.$$

$$x = \pm 2, \qquad y = 15 - 4 - \frac{16}{4} = 7.$$

The stationary points are $(-2, 7)$ and $(2, 7)$. *(3)*

(b) Use the second derivative test.

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2 - 96x^{-4}.$$

At either stationary point this is $-2 - 96/16 = -8 < 0$, so the curve is concave down there.

Both stationary points are maximum points. *(2)*

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Question 6

Question 6

(a) Show that $\sin\left(x+\frac{\pi}{6}\right)=\cos\left(x-\frac{\pi}{4}\right)$ can be written as $\tan x=\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{2}}$. (b) Hence solve the equation for $0\leq x\leq 2\pi$.

Model solution

(a) Expand both compound angles, then collect the sine and cosine terms.

$$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x=\frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\sin x.$$

$$\left(\sqrt{3}-\sqrt{2}\right)\sin x=\left(\sqrt{2}-1\right)\cos x.$$

Dividing by $\left(\sqrt{3}-\sqrt{2}\right)\cos x$ *gives*

$$\tan x=\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{2}}.$$

Required form shown. **(4)**

(b) The right-hand side is positive, so the two solutions in one full cycle are in quadrants I and III.

$$x=\arctan\left(\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{2}}\right),\quad \pi+\arctan\left(\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{2}}\right).$$

$x=\frac{7\pi}{24}$ *or* $\frac{31\pi}{24}$. **(1)**

Question 7

Question 7

Determine the exact distance between the two points at which the line through $(4, 5)$ and $(6, -1)$ meets the curve $y = 2x^2 - 7x + 1$.

Model solution

Find the line first, then solve its intersection with the curve.

$$m = \frac{-1-5}{6-4} = -3, \qquad y = -3x + 17.$$

$$-3x + 17 = 2x^2 - 7x + 1 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x = -2, 4.$$

$$y = 23 \text{ when } x = -2, \qquad y = 5 \text{ when } x = 4.$$

The intersection points are $(-2, 23)$ *and* $(4, 5)$*, so use Pythagoras.*

$$d = \sqrt{(4+2)^2 + (5-23)^2} = \sqrt{36 + 324} = 6\sqrt{10}.$$

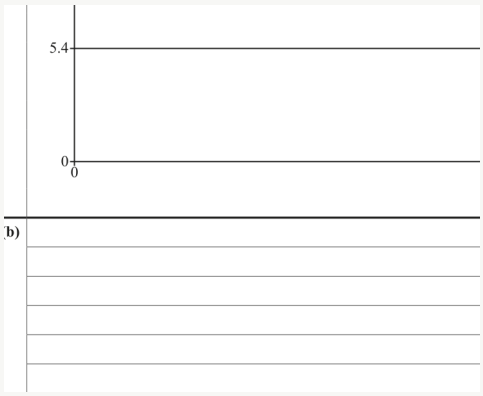
$6\sqrt{10}$. *(7)*

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Question 8

Question 8

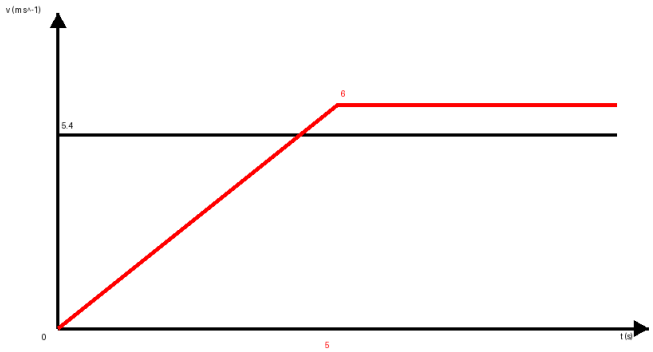
A bus travels along a straight road at 5.4 m s^{-1}. At $t = 0$, as it passes a boy, the boy accelerates from rest at 1.2 m s^{-2} for 5 s. He then runs at constant speed until he catches the bus.



(a) Draw the boy's velocity–time graph on the given axes.
(b) Determine when the boy has the same speed as the bus. (c) Find their maximum separation.
(d) Find the distance run when he catches the bus.

Model solution

(a) Constant acceleration gives a straight line from $(0, 0)$ to $(5, 6)$; thereafter speed is constant, so the graph is horizontal at $v = 6$.



Completed velocity--time graph. **(3)**

(b) During acceleration, $v = 1.2t$.

$$1.2t = 5.4 \Rightarrow t = 4.5.$$

4.5 *s.* **(2)**

(c) Separation is greatest when the relative velocity changes from positive to negative, i.e. at $t = 4.5$. It is the area between the graphs.

$$\frac{1}{2}(4.5)(5.4) = 12.15.$$

12.15 *m.* **(3)**

(d) At 5 s the boy has run $\frac{1}{2}(5)(6) = 15$ m, while the bus has run $5.4(5) = 27$ m. The 12 m gap

closes at relative speed $6 - 5.4 = 0.6$ *m s*$^{-1}$.

$$t_{\text{extra}} = \frac{12}{0.6} = 20 \text{ s}, \quad T = 25 \text{ s}, \quad s = 5.4(25) = 135 \text{ m}.$$

135 *m*. *(3)*

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Question 9

Question 9

The gradient of a curve is $\frac{\mathrm{d}y}{\mathrm{d}x} = e^x - 4e^{-x}$. (a) Show that gradient 3 gives $(e^x)^2 - 3e^x - 4 = 0$. (b) Hence show there is only one such point, stating its exact x-coordinate. (c) The curve passes through $(0, 0)$. Show that when $x = 1$ the curve is below the x-axis.

Model solution

(a) Set the given gradient equal to 3, then multiply by e^x.

$$e^x - 4e^{-x} = 3 \Rightarrow e^{2x} - 3e^x - 4 = 0.$$

$(e^x)^2 - 3e^x - 4 = 0.$ *(2)*

(b) Treat e^x as the unknown.

$$(e^x - 4)(e^x + 1) = 0.$$

Since $e^x > 0$, $e^x = -1$ is impossible.

$x = \ln 4$, *the only value.* *(3)*

(c) Integrate the gradient, then use the point to find the constant.

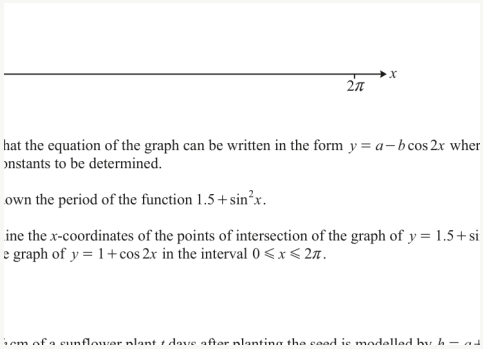
$$y = e^x + 4e^{-x} + c, \qquad 0 = 1 + 4 + c \Rightarrow c = -5.$$

$$y(1) = e + \frac{4}{e} - 5 \approx -0.810 < 0.$$

The curve is below the x-axis at $x = 1$. *(5)*

Question 10

Question 10



The diagram shows $y = 1.5 + \sin^2 x$, for $0 \leq x \leq 2\pi$. ...om of a sunflower plant t days after planting the seed is modelled by $h = a +$

(a) Show it can be written as $y = a - b\cos 2x$. (b) Write down the period. (c) Determine the x-coordinates where it intersects $y = 1 + \cos 2x$ in this interval.

Model solution

(a) Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.

$$y = 1.5 + \frac{1}{2}(1 - \cos 2x) = 2 - \frac{1}{2}\cos 2x.$$

$a = 2,\ b = \frac{1}{2}$. ***(2)***

(b) The argument $2x$ ***doubles frequency.***

Period $= \pi$. ***(1)***

(c) Equate the functions, using the form found in part (a).

$$2 - \frac{1}{2}\cos 2x = 1 + \cos 2x \Rightarrow \cos 2x = \frac{2}{3}.$$

$x \approx 0.421,\ 2.72,\ 3.56,\ 5.86$ ***radians.*** ***(3)***

Question 11

Question 11

The height h cm of a sunflower t days after planting is modelled by $h = a + b\ln t$, for $t \geq 9$. It is 10 cm at $t = 10$ and 200 cm at $t = 85$. (a) (i) Show $b = 88.8$, correct to 3 significant figures. (ii) Find a. (b) Explain why the model is unsuitable for (i) small positive t, (ii) very large positive t. (c) Show it reaches 1 m in less than half the time it takes to reach 2 m. (d) Find t when growth rate is 3 cm per day.

Model solution

(a)(i) Substitute both data points and eliminate a.

$$10 = a + b\ln 10, \quad 200 = a + b\ln 85, \quad 190 = b\ln(8.5).$$

$$b = \frac{190}{\ln 8.5} = 88.8 \text{ (3 s.f.)}.$$

$b = 88.8$. **(2)**

(a)(ii) Substitute back.

$$a = 10 - 88.8\ln 10 \approx -194.$$

$a \approx -194$. **(1)**

(b)(i) Small t makes $\ln t$ negative and can predict a negative height.

Negative height is not physically possible. **(1)**

(b)(ii) As t grows, $\ln t$ grows without bound.

It predicts indefinite growth, which is unrealistic. **(1)**

(c) Half of 85 days is 42.5 days. Evaluate the model then.

$$h(42.5) \approx -194 + 88.8\ln 42.5 \approx 139 \text{ cm} > 100 \text{ cm}.$$

It reaches 1 m in less than 42.5 days, hence in less than half the time. **(2)**

(d) Differentiate the model and set the rate equal to 3.

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{88.8}{t} = 3 \Rightarrow t = 29.6.$$

$t \approx 29.6$ *days.* **(3)**

Question 12

Question 12

Unit vectors $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertically upwards. A particle has mass 2 kg. (a) Write down its weight. A force $3\mathbf{i}$ N and $\mathbf{F} = (-4\mathbf{i} + 12\mathbf{j})$ N act on it. (b) Determine its acceleration. (c) Its initial velocity is $5\mathbf{i}$ m s^{-1}. Find its velocity after 4 s. (d) Find the extra force needed for constant velocity.

Model solution

(a) $-2g\mathbf{j} = -19.6\mathbf{j}$ *N.* *(1)*

(b) Apply Newton's second law to the vector resultant.

$$(-4\mathbf{i} + 12\mathbf{j}) + 3\mathbf{i} - 19.6\mathbf{j} = 2\mathbf{a}.$$

$\mathbf{a} = -0.5\mathbf{i} - 3.8\mathbf{j}$ *m s*$^{-2}$. *(3)*

(c) Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$.

$$\mathbf{v} = 5\mathbf{i} + 4(-0.5\mathbf{i} - 3.8\mathbf{j}).$$

$\mathbf{v} = 3\mathbf{i} - 15.2\mathbf{j}$ *m s*$^{-1}$. *(2)*

(d) Constant velocity means zero resultant force, so add the negative of the current resultant.

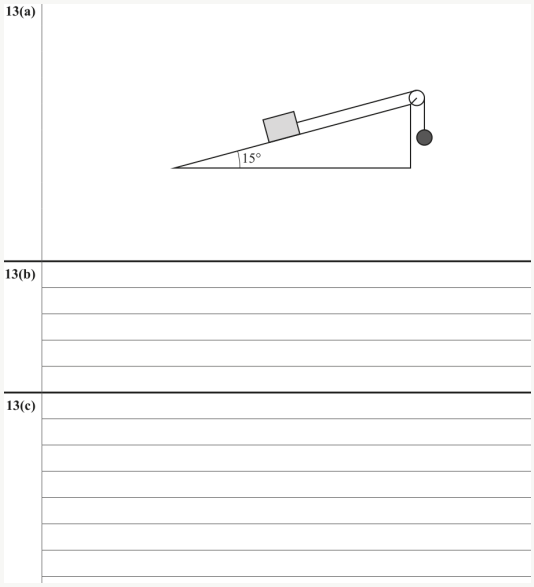
$\mathbf{F}_{\text{extra}} = \mathbf{i} + 7.6\mathbf{j}$ *N.* *(1)*

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Question 13

Question 13

A block of mass 8 kg is on a rough plane inclined at $15°$, with coefficient of friction 0.3. It is connected over a smooth pulley to a hanging sphere of mass 5 kg. Released from rest, the sphere falls and the block accelerates up the plane at a m s^{-2}.



(a) Show all forces on the block and sphere.
(b) Write the equation of motion for the sphere. (c) Determine a.

Model solution

(a) The block has tension T up the plane, friction $0.3R$ and component of weight down the plane, normal reaction R perpendicular to the plane, and weight $8g$ vertically down. The sphere has T upward and $5g$ downward.

Forces as stated, with friction down the plane. **(4)**

(b) Choose downwards positive for the falling sphere.

$5g - T = 5a.$ **(2)**

(c) For the block, perpendicular equilibrium gives $R = 8g\cos 15°$, so friction is $0.3R = 2.4g\cos 15°$. Take up the plane as positive.

$$T - 8g\sin 15° - 2.4g\cos 15° = 8a.$$

Add this to $5g - T = 5a$ to eliminate the tension.

$$5g - 8g\sin 15° - 2.4g\cos 15° = 13a.$$

$a = 0.461$ *m s^{-2} (3 s.f.).* **(6)**

Question 14

Question 14

(a) Use logarithm laws to show $\log_{10} 200 - \log_{10} 20 = 1$. The first three terms of a sequence are $\log_{10} 20, \log_{10} 200, \log_{10} 2000$. (b) Show it is arithmetic. (c) Find the exact sum of its first 50 terms.

Model solution

(a) The difference law changes a subtraction of logarithms into a logarithm of a quotient.

$$\log_{10} 200 - \log_{10} 20 = \log_{10} \left(\frac{200}{20} \right) = \log_{10} 10 = 1.$$

Shown. *(2)*

(b) The next difference is the same.

$$\log_{10} 2000 - \log_{10} 200 = \log_{10} 10 = 1.$$

The common difference is 1, so the sequence is arithmetic. *(2)*

(c) Use $S_n = \frac{n}{2}[2a + (n-1)d]$, with $a = \log_{10} 20$ and $d = 1$.

$$S_{50} = 25 \left(2 \log_{10} 20 + 49 \right).$$

$25 \left(2 \log_{10} 20 + 49 \right).$ *(2)*

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Question 15

Question 15

A projectile is launched from level ground with initial speed u, at angle θ above the horizontal. (a) Show that its range is $\dfrac{2u^2 \sin\theta\cos\theta}{g}$. (b) Determine the acute values of θ for which the maximum height is greater than the range. Give degrees.

Model solution

(a) Resolve the initial velocity. Vertical motion returns to the launch level, so $s = 0$.

$$0 = u\sin\theta\, t - \frac{1}{2}gt^2 \Rightarrow t = \frac{2u\sin\theta}{g}.$$

Horizontal speed remains $u\cos\theta$, hence

$$R = u\cos\theta\left(\frac{2u\sin\theta}{g}\right) = \frac{2u^2\sin\theta\cos\theta}{g}.$$

Required result shown. **(3)**

(b) At maximum height the vertical velocity is zero.

$$0 = (u\sin\theta)^2 - 2gH \Rightarrow H = \frac{u^2\sin^2\theta}{2g}.$$

$$\frac{u^2\sin^2\theta}{2g} > \frac{2u^2\sin\theta\cos\theta}{g} \Rightarrow \tan\theta > 4.$$

$76.0° < \theta < 90°$. **(5)**