Maths with Matt

Physics and maths resources

OCR A Level Mathematics B (MEI)

H640/01 - Pure Mathematics and Mechanics

Wednesday 4 June 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Wednesday 4 June 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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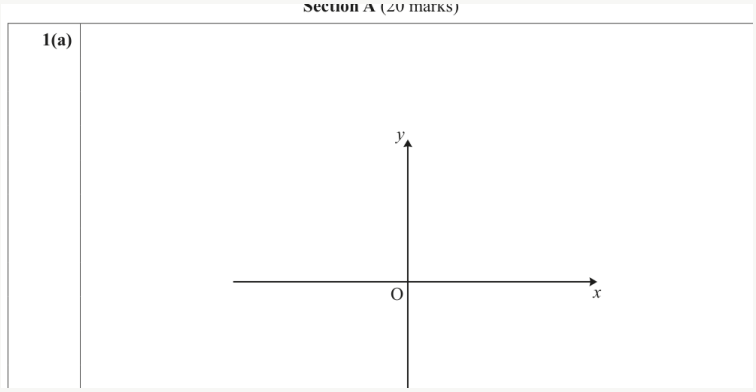
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Question 1

Question 1

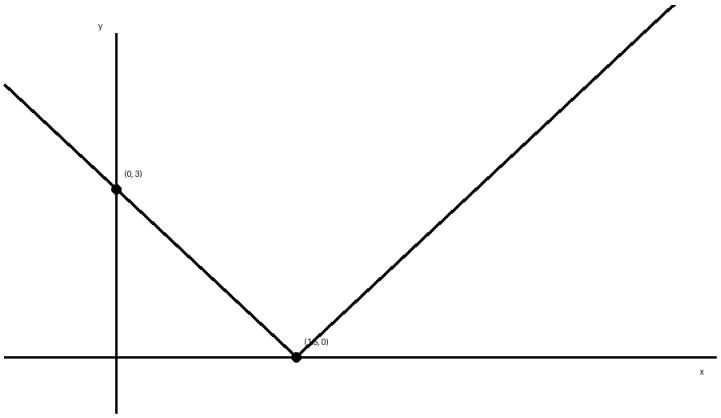
(a) Sketch the function $y = |2x - 3|$.



(b) In this question you must show detailed reasoning. Solve the equation $|2x - 3| = 4 - x$.

Model solution

(a) The absolute value makes a V-shaped graph. The expression inside the modulus is zero at $x = \frac{3}{2}$, which gives the vertex; its value at $x = 0$ gives a second useful point.



A V-shaped graph with vertex $\left(\frac{3}{2}, 0\right)$, passing through $(0, 3)$. **(2)**

(b) Since the right-hand side must be non-negative, and an absolute value has two possible signed values, solve both linear equations.

$$2x - 3 = 4 - x \quad \textbf{or} \quad 2x - 3 = -(4 - x).$$

$$3x = 7 \implies x = \frac{7}{3}, \qquad x - 3 = -4 \implies x = -1.$$

$x = -1$ **or** $x = \frac{7}{3}$. **(3)**

Question 2

Question 2

Express $\dfrac{7x-25}{(x-1)(x-4)^2}$ in partial fractions.

Model solution

Because $x-4$ is a repeated linear factor, a term is needed for each power of that factor.

$$\frac{7x-25}{(x-1)(x-4)^2} = \frac{A}{x-1} + \frac{B}{x-4} + \frac{C}{(x-4)^2}.$$

$$7x-25 = A(x-4)^2 + B(x-1)(x-4) + C(x-1).$$

Choosing roots of the factors isolates constants efficiently.

$$x=4: \quad 3=3C \implies C=1, \qquad x=1: \quad -18=9A \implies A=-2.$$

$$0 = A+B \implies B=2.$$

$$\frac{7x-25}{(x-1)(x-4)^2} = -\frac{2}{x-1} + \frac{2}{x-4} + \frac{1}{(x-4)^2}. \qquad \textit{(4)}$$

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Question 3

Question 3

(a) Evaluate $\sum_{r=1}^{4} r^{-1}$, giving your answer as a fraction in its lowest terms.
(b) Write the sum $1 + 3 + 5 + 7 + 9$ in a similar way to the series in part (a).
(c) Explain why the sum to infinity of $1 + 3 + 5 + 7 + 9 + \ldots$ is not well defined.

Model solution

(a) Expand the finite sum, then use a common denominator.

$$\sum_{r=1}^{4} r^{-1} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{12 + 6 + 4 + 3}{12} = \frac{25}{12}.$$

$\frac{25}{12}$. *(1)*

(b) The rth positive odd number is $2r - 1$. Five terms therefore require $r = 1$ to $r = 5$.

$\sum_{r=1}^{5}(2r - 1)$. *(2)*

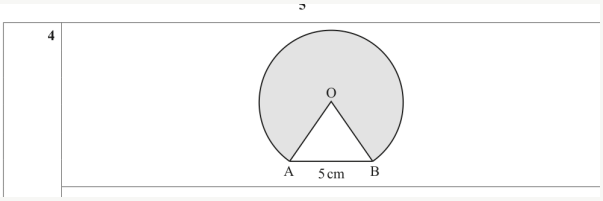
(c) Every added term is positive, so the partial sums continually increase without bound. They do not approach a finite number.

The infinite sum diverges, so it is not well defined. *(1)*

Question 4

Question 4

The diagram shows part of a circle with centre O and radius 5 cm. The circle passes through points A and B, and $AB = 5$ cm. Calculate the area of the shaded region.



Model solution

Since $OA = OB = AB = 5$, triangle OAB is equilateral. The shaded region is the major sector, so use its reflex angle.

$$\angle AOB = \frac{\pi}{3}, \qquad \theta_{\text{major}} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}.$$

$$\textit{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}(5)^2\left(\frac{5\pi}{3}\right) = \frac{125\pi}{6}\ \textit{cm}^2.$$

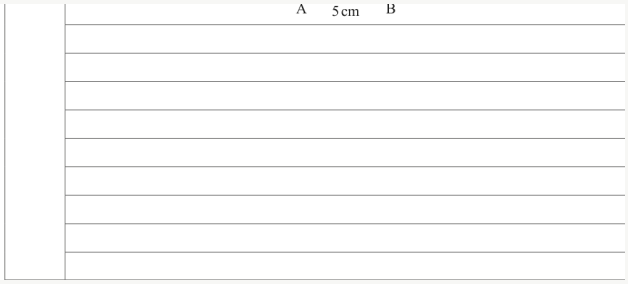
$\frac{125\pi}{6}$ *cm*^{2} $(\approx 65.4$ *cm*$^2)$. **(4)**

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Question 5

Question 5

A uniform rectangular lamina $ABCD$ has mass 0.12 kg, with $AB = 11$ cm and $AD = 20$ cm. It is held by a smooth hinge at A. A horizontal force of magnitude P N is applied at B so that the lamina is in equilibrium in a vertical plane with AD horizontal, as shown. Show that P is 1.07 to 3 significant figures.



Model solution

Take moments about the hinge: this removes the unknown hinge reaction. The weight acts at the centre, which is 10 cm horizontally from A; the horizontal force at B has perpendicular distance 11 cm.

$$(0.12g)(0.10) = P(0.11).$$

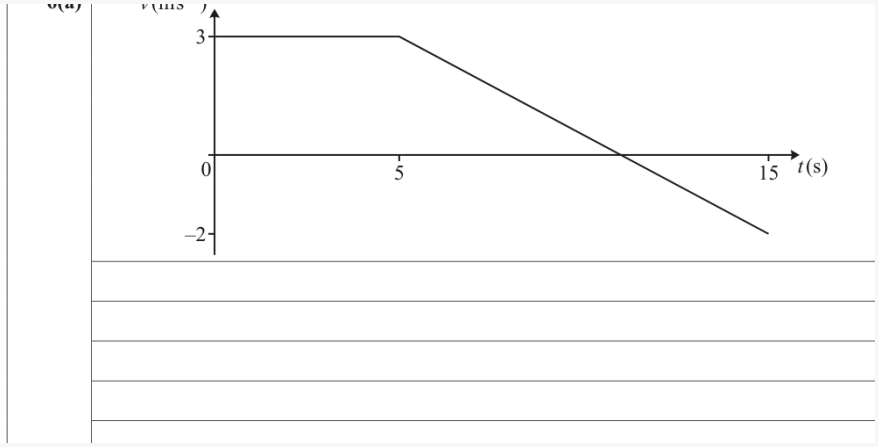
$$P = \frac{0.12(9.8)(0.10)}{0.11} = 1.06909\ldots$$

$P = 1.07$ **N (3 s.f.).** **(3)**

Question 6

Question 6

A car travels along a straight horizontal road. Its velocity v m s^{-1} at time t s is shown on the velocity–time graph: $(0, 3)$ and $(5, 3)$ are joined, and $(5, 3)$ and $(15, -2)$ are joined.



(a) Calculate the total distance travelled in the first 15 s.
The car is then attached to a caravan by a light inextensible horizontal tow bar. Both accelerate at 1.5 m s^{-2}. The masses are 1400 kg and 900 kg; the driving force on the car is D N and tow-bar tension is T N. Resistances are 400 N on the car and 450 N on the caravan.
(b) Write down the equations of motion for the car and caravan separately.
(c) Calculate D and T.

Model solution

(a) Distance is the total area between a velocity--time graph and the time axis. The velocity reaches zero at $t = 11$, so the final triangular area is also counted positively.

$$\textit{distance} = 5(3) + \frac{1}{2}(6)(3) + \frac{1}{2}(4)(2) = 15 + 9 + 4 = 28.$$

28 *m.* *(3)*

(b) Take forward as positive. Apply Newton's second law separately to each body.

$$\textit{car: } D - T - 400 = 1400(1.5), \qquad \textit{caravan: } T - 450 = 900(1.5).$$

$D - T - 400 = 2100$, $T - 450 = 1350$. *(2)*

(c) First use the caravan equation to find the tension, then use the combined system to eliminate it.

$$T = 1350 + 450 = 1800 \text{ N},$$

$$D - 850 = (1400 + 900)(1.5) = 3450 \implies D = 4300 \text{ N}.$$

$D = 4300$ N, $T = 1800$ N. *(3)*

Question 7

Question 7

In this question you must show detailed reasoning. Show that the area of the finite region enclosed by $y = x^2 - 7x + 2$ and $y = 14 - 9x - x^2$ is $\frac{125}{3}$.

Model solution

The limits of the enclosed region are where the curves intersect. Subtracting the equations also identifies which curve is above between those limits.

$$x^2 - 7x + 2 = 14 - 9x - x^2 \implies 2x^2 + 2x - 12 = 0 \implies (x+3)(x-2) = 0.$$

$$x = -3 \text{ or } x = 2.$$

For example at $x = 0$, $14 - 9x - x^2$ is above $x^2 - 7x + 2$. Hence

$$\begin{aligned} \text{Area} &= \int_{-3}^{2} \left[(14 - 9x - x^2) - (x^2 - 7x + 2)\right] \mathrm{d}x \\ &= \int_{-3}^{2} (12 - 2x - 2x^2) \, \mathrm{d}x \\ &= \left[12x - x^2 - \frac{2}{3}x^3\right]_{-3}^{2} \\ &= \left(24 - 4 - \frac{16}{3}\right) - (-36 - 9 + 18) = \frac{125}{3}. \end{aligned}$$

$\frac{125}{3}$ *square units.* *(7)*

Question 8

Question 8

(a) Determine the two values of k for which $y = 3x + 5$ is a tangent to $y = k - kx - x^2$.
(b) These values define two curves of the form $y = k - kx - x^2$. Determine the coordinates of their point of intersection.

Model solution

(a) A tangent gives one repeated intersection, so after equating the equations the quadratic in x has discriminant zero.

$$3x + 5 = k - kx - x^2 \implies x^2 + (k+3)x + (5-k) = 0.$$

$$(k+3)^2 - 4(5-k) = 0 \implies k^2 + 10k - 11 = 0 \implies (k+11)(k-1) = 0.$$

$k = -11$ **or** $k = 1$. **(4)**

(b) Substitute both values of k and equate the two curve equations.

$$-11 + 11x - x^2 = 1 - x - x^2 \implies 12x = 12 \implies x = 1.$$

$$y = 1 - 1 - 1 = -1.$$

$(1, -1)$. **(3)**

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Question 9

Question 9

The table gives data for four moons of Jupiter.

Moon	Semi-major axis (km)	Orbital period (Earth days)
Io	421 800	1.7627
Europa	671 100	3.5255
Ganymede	1 070 400	7.1556
Callisto	1 882 700	16.690

A student models orbital period T in Earth days and semi-major axis d in km by $T = kd^n$, where k and n are constants.
(a) Show that $\log_{10} T = \log_{10} k + n \log_{10} d$.
A plot of $\log_{10} T$ against $\log_{10} d$ has gradient 1.504 and $\log_{10} T$-axis intercept -8.215.
(b) Determine consistent values of k and n.
(c) Thebe has $d = 221\,900$ km and an online encyclopedia gives its period as 0.6761 Earth days. Comment on the suitability of the model for Thebe.

Model solution

(a) Taking logarithms turns the power model into a straight-line form.

$$\log_{10} T = \log_{10}(kd^n) = \log_{10} k + \log_{10}(d^n) = \log_{10} k + n \log_{10} d.$$

$\log_{10} T = \log_{10} k + n \log_{10} d.$ *(1)*

(b) Compare with $y = c + mx$: the gradient is n, and the intercept is $\log_{10} k$.

$$n = 1.504, \qquad \log_{10} k = -8.215,$$

$$k = 10^{-8.215} = 6.095\ldots \times 10^{-9}.$$

$n = 1.504, \quad k \approx 6.10 \times 10^{-9}.$ *(3)*

(c) Use the fitted model at Thebe's semi-major axis, then compare rather than expecting an exact match from a model based on a line of best fit.

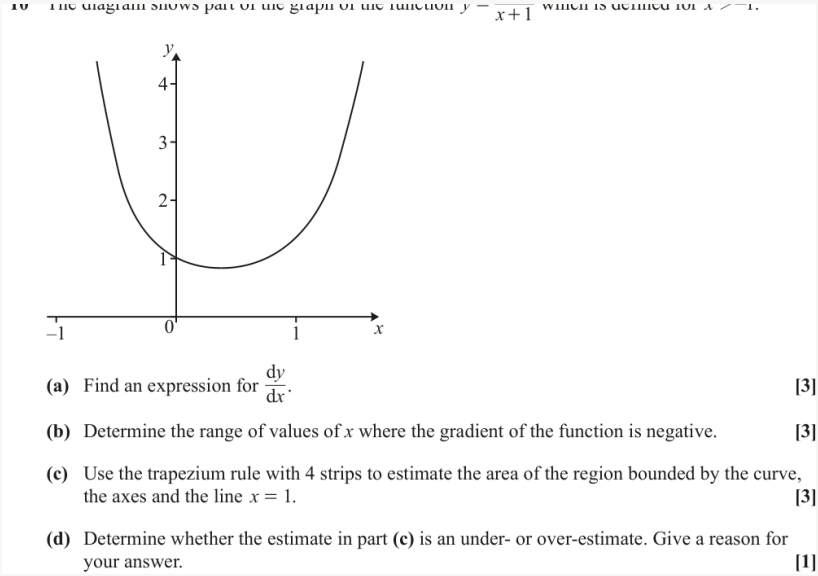
$$T = (6.095\ldots \times 10^{-9})(221\,900)^{1.504} = 0.669\ldots \text{ days}.$$

The prediction is close to 0.6761 days, so the model is suitable for Thebe. *(2)*

Question 10

Question 10

The diagram shows part of the graph of $y = \dfrac{e^{x^2}}{x+1}$, defined for $x > -1$.



(a) Find $\dfrac{dy}{dx}$.
(b) Determine the range of x for which the gradient is negative.
(c) Use the trapezium rule with 4 strips to estimate the area bounded by the curve, the axes and $x = 1$.
(d) Determine whether the estimate in part (c) is an under- or over-estimate. Give a reason.

Model solution

(a) This is a quotient, so use the quotient rule. The chain rule gives $\frac{d}{dx}e^{x^2} = 2xe^{x^2}$.

$$\frac{dy}{dx} = \frac{(x+1)(2xe^{x^2}) - e^{x^2}}{(x+1)^2} = \frac{e^{x^2}(2x^2+2x-1)}{(x+1)^2}.$$

$$\frac{dy}{dx} = \frac{e^{x^2}(2x^2+2x-1)}{(x+1)^2}. \qquad \textit{(3)}$$

(b) The exponential and squared denominator are positive in the domain, so only the quadratic numerator determines the sign.

$$2x^2 + 2x - 1 < 0.$$

$$x = \frac{-2 \pm \sqrt{4+8}}{4} = \frac{-1 \pm \sqrt{3}}{2}.$$

The upward-opening quadratic is negative between its roots; the left root lies outside $x > -1$.

$$-1 < x < \frac{-1+\sqrt{3}}{2} \ (\approx 0.366). \qquad \textit{(3)}$$

(c) Four equal strips from 0 to 1 give width $h = 0.25$.

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.851596	0.856017	1.002888	1.359141

$$A \approx \frac{0.25}{2}\left[f(0) + 2\{f(0.25) + f(0.5) + f(0.75)\} + f(1)\right]$$
$$= 0.125(7.78014\ldots) = 0.9725\ldots$$

Area ≈ 0.9725 square units. *(3)*

(d) The curve is concave upwards on this interval, so each trapezium chord lies above the curve.

Over-estimate. *(1)*

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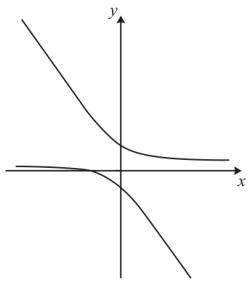
Question 11

Question 11

The curve is defined implicitly by $(2x + y)(y - 1) = 6$.

your answer. [1]

11 The curve in the graph below is defined implicitly by $(2x+y)(y-1) = 6$.



(a) Show that $\frac{dy}{dx} = \frac{2(1-y)}{2y+2x-1}$. [4]

(a) Show that $\dfrac{dy}{dx} = \dfrac{2(1-y)}{2y+2x-1}$.
(b) Find the equation of the normal at $(2, 2)$.
(c) Show that there is no point on the curve where the tangent is parallel to the x-axis.

Model solution

(a) Both bracketed factors depend on x, because $y = y(x)$. Apply the product rule and implicit differentiation.

$$\left(2 + \frac{dy}{dx}\right)(y - 1) + (2x + y)\frac{dy}{dx} = 0.$$

$$(2y + 2x - 1)\frac{dy}{dx} + 2y - 2 = 0,$$

$$\frac{dy}{dx} = \frac{2(1 - y)}{2y + 2x - 1}.$$

$$\frac{dy}{dx} = \frac{2(1 - y)}{2y + 2x - 1}.$$ *(4)*

(b) Evaluate the tangent gradient first. A normal has the negative reciprocal gradient.

$$\left.\frac{dy}{dx}\right|_{(2,2)} = \frac{2(1 - 2)}{4 + 4 - 1} = -\frac{2}{7}, \qquad m_{\text{normal}} = \frac{7}{2}.$$

$$y - 2 = \frac{7}{2}(x - 2).$$ *(3)*

(c) A tangent parallel to the x-axis would have gradient zero. The derivative formula then requires its numerator to be zero.

$$2(1 - y) = 0 \implies y = 1.$$

$$(2x + 1)(1 - 1) = 6 \implies 0 = 6,$$

which is impossible.

There is no such point on the curve. **(3)**

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Question 12

Question 12

In this question x and y are horizontal and upwards vertical directions respectively. An astronaut is on the moon.

(a) A ball is dropped from rest from 1 m above the surface and takes 1.1 s to hit it. Calculate the acceleration due to the moon's gravity, to 3 significant figures.

A ball is hit from the moon's surface with speed 25 m s^{-1}, at $40°$ above the horizontal in the positive x-direction.

(b) Taking the origin as the point of projection, determine the trajectory $y = f(x)$, with coefficients to 3 significant figures.

(c) The crater edge is 40 m from the point of projection and 15 m above it. Determine whether the ball goes over the edge.

Model solution

(a) Take upwards as positive. The displacement is -1 m and the initial velocity is zero.

$$-1 = 0(1.1) + \frac{1}{2}a(1.1)^2 \implies a = -1.65289\ldots$$

$a = -1.65$ *m s^{-2} (downwards).* *(2)*

(b) Resolve the initial velocity. Horizontal acceleration is zero, allowing time to be eliminated from the vertical equation.

$$x = 25\cos 40°\, t \implies t = \frac{x}{25\cos 40°},$$

$$y = 25\sin 40°\, t + \frac{1}{2}(-1.65)t^2.$$

$$y = 25\sin 40°\left(\frac{x}{25\cos 40°}\right) - 0.825\left(\frac{x}{25\cos 40°}\right)^2 = 0.839x - 0.00225x^2.$$

$y = 0.839x - 0.00225x^2$. *(4)*

(c) Substitute the horizontal position of the edge into the trajectory.

$$y(40) = 0.839(40) - 0.00225(40)^2 = 30.0 \text{ m}.$$

Yes. At $x = 40$, the ball is at 30.0 m, which is above 15 m. *(2)*

Question 13

Question 13

The displacement **r** m of a parachutist t s after opening their parachute is modelled by

$$\mathbf{r} = \begin{pmatrix} 50t \\ 5t + 280\left(1 - e^{-0.16t}\right) \end{pmatrix},$$

where the x-direction is horizontal and the y-direction is vertically downwards.
(a) Calculate the distance from the parachutist's initial position after 10 s.
(b) Find a vector expression for velocity.
(c) Determine the velocity predicted for large t.
(d) Parachutists usually land travelling approximately vertically. Explain a factor that should be included to improve the model.

Model solution

(a) Substitute $t = 10$, then take the magnitude of the displacement vector.

$$\mathbf{r}(10) = \begin{pmatrix} 500 \\ 50 + 280(1 - e^{-1.6}) \end{pmatrix} = \begin{pmatrix} 500 \\ 273.472\ldots \end{pmatrix}.$$

$$|\mathbf{r}(10)| = \sqrt{500^2 + (273.472\ldots)^2} = 569.9\ldots$$

570 *m (or* 569.9 *m).* *(3)*

(b) Velocity is the derivative of displacement, differentiating each component.

$$\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \begin{pmatrix} 50 \\ 5 + 44.8e^{-0.16t} \end{pmatrix}.$$

$\mathbf{v} = \begin{pmatrix} 50 \\ 5 + 44.8e^{-0.16t} \end{pmatrix}$ *m s*$^{-1}$. *(3)*

(c) As t becomes large, $e^{-0.16t}$ tends to zero.

$\mathbf{v} \to \begin{pmatrix} 50 \\ 5 \end{pmatrix}$ *m s*$^{-1}$. *(2)*

(d) The constant horizontal component incorrectly predicts substantial sideways motion at landing.

Include air resistance with a horizontal component. *(1)*

Question 14

Question 14

The **i** and **j** vectors are horizontal and vertically upwards. A particle of mass 5 kg rests on a rough horizontal shelf; its coefficient of friction is μ.
(a) A force $\mathbf{P} = 9\mathbf{i} + 20\mathbf{j}$ N acts. The particle is on the point of sliding along the shelf. Determine μ.
The force **P** is removed and one end of the shelf is lifted to make an angle $\alpha°$ with the horizontal.
(b) The particle is on the point of sliding down the shelf. Show that $\alpha = 17.2°$ to 3 significant figures.
(c) The particle is projected up the shelf at 5 m s^{-1}. Given it remains in contact with the shelf, determine when it first comes to rest.

Model solution

(a) The impending motion is horizontal, so limiting friction opposes the 9 N horizontal component. Resolve equilibrium horizontally and vertically.

$$F = 9, \qquad R + 20 - 5g = 0 \implies R = 49 - 20 = 29 \text{ N}.$$

$$F = \mu R \implies \mu = \frac{9}{29} = 0.310\ldots$$

$\mu = \dfrac{9}{29} \approx 0.310.$ **(5)**

(b) On the inclined shelf, resolve perpendicular and parallel to the shelf. At limiting equilibrium, the downslope weight component equals limiting friction.

$$R = 5g\cos\alpha, \qquad 5g\sin\alpha = \mu R = \frac{9}{29}(5g\cos\alpha).$$

$$\tan\alpha = \frac{9}{29} \implies \alpha = \tan^{-1}\left(\frac{9}{29}\right) = 17.2°.$$

$\alpha = 17.2°$ ***(3 s.f.).*** **(4)**

(c) Take up the slope as positive. While the particle moves up, both the weight component and friction act down the slope. Newton's second law therefore gives a negative acceleration.

$$-\left(5g\sin 17.2° + \frac{9}{29}(5g\cos 17.2°)\right) = 5a.$$

$$a = -5.80\ldots \text{ m s}^{-2}.$$

Use $v = u + at$ at the first instant the particle stops.

$$0 = 5 + (-5.80\ldots)t \implies t = 0.862\ldots \text{ s}.$$

$t = 0.862$ ***s.*** **(6)**