Maths with Matt

Physics and maths resources

OCR A Level Mathematics B (MEI)

H640/02 - Pure Mathematics and Statistics

Tuesday 14 June 2022

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Tuesday 14 June 2022 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are exact values to be determined.
(4)

Model solution

Equate coefficients after expanding the required form.

$$R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha.$$

$$R\cos\alpha = 1, \quad R\sin\alpha = \sqrt{3}, \quad R^2 = 4.$$

Thus $R = 2$, $\tan\alpha = \sqrt{3}$, *and* $\alpha = \pi/3$.

$2\cos(\theta - \frac{\pi}{3})$. ***(4)***

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Question 2

Question 2

Find the sum of the infinite series $50 + 25 + 12.5 + 6.25 + \cdots$. **(2)**

Model solution

This is geometric with $a = 50$ *and* $r = 0.5$.

$$S_\infty = \frac{50}{1 - 0.5} = 100.$$

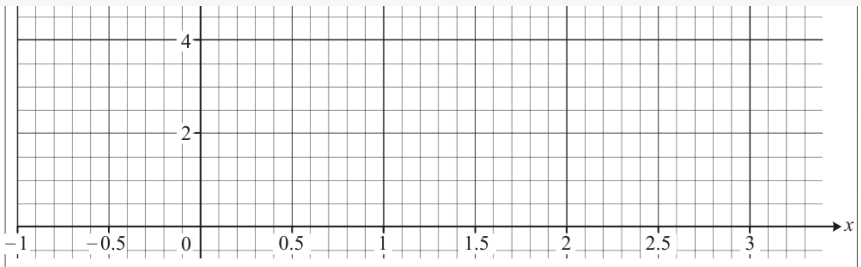
$100.$ *(2)*

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Question 3

Question 3

(**a**) On the axes below, sketch the curve with equation $y = 3(0.4)^x$. **(3)**



3(b)

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Turn over

(**b**) Given that $3(0.4)^x = 0.8$, determine x correct to 3 significant figures. **(3)**

Model solution

(a) The base is between zero and one, so draw decreasing exponential decay through $(0, 3)$, above the axis, with asymptote $y = 0$. (b) Take logarithms to solve for the exponent.

$$x = \frac{\log(0.8/3)}{\log(0.4)} = 1.4425\ldots$$

$x = 1.44$ *(3 s.f.).* **(3)**

Question 4

Question 4

A survey of university students revealed that:

- 31% have a part-time job but do not play competitive sport.
- 23% play competitive sport but do not have a part-time job.
- 22% do not play competitive sport and do not have a part-time job.

(**a**) Show this information on a Venn diagram. **(2)**
A student is selected at random.
(**b**) Determine the probability that the student plays competitive sport and has a part-time job. **(2)**

Model solution

Let J be part-time job and S competitive sport. The non-overlap regions are $P(J \cap S') = 0.31$, $P(J' \cap S) = 0.23$, and $P(J' \cap S') = 0.22$.

$$P(J \cap S) = 1 - 0.31 - 0.23 - 0.22 = 0.24.$$

Place 24% in the overlap; the required probability is 0.24. ***(2)***

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Question 5

Question 5

Tom conjectures that if n is an odd number greater than 1, then $2^n - 1$ is prime.
Find a counterexample to disprove Tom's conjecture. **(3)**

Model solution

Try an odd value greater than one and show its result is composite.

$$2^9 - 1 = 511 = 7 \times 73.$$

$n = 9$, *since* 511 *is not prime.* ***(3)***

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Question 6

Question 6

X is continuous with $X \sim N(\mu, \sigma^2)$. On the sketch of this Normal distribution in the Printed Answer Booklet, shade the area bounded by the curve, the x-axis and the lines $x = \mu \pm \sigma$. **(2)**

Model solution

The points one standard deviation from the mean are the inflection points.

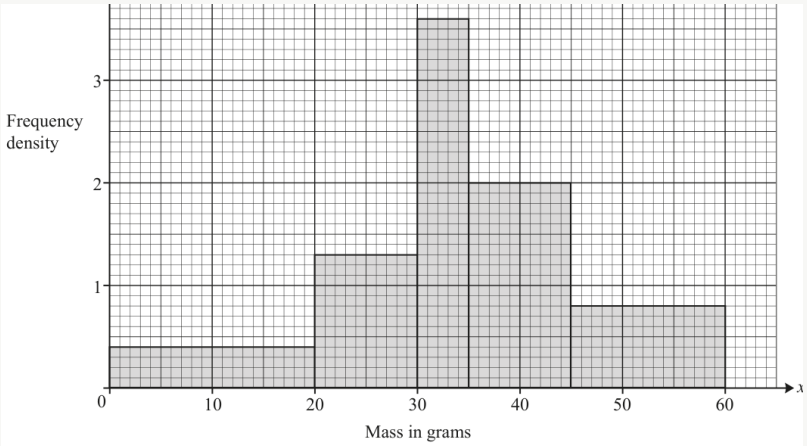
Shade the central region $\mu - \sigma \leq x \leq \mu + \sigma$. *(2)*

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Question 7

Question 7

Kareem bought some tomatoes. He recorded the mass of each tomato and displayed the results in a histogram, shown below.



Determine how many tomatoes Kareem bought. [2]

Determine how many tomatoes Kareem bought. **(2)**

Model solution

Frequency is the area of a histogram bar.

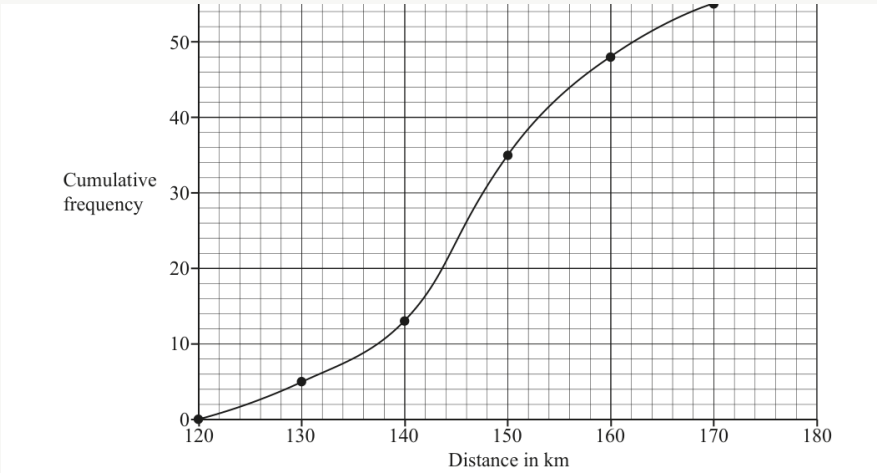
$$0.4(20) + 1.3(10) + 3.6(5) + 2(10) + 0.8(15) = 8 + 13 + 18 + 20 + 12 = 71.$$

71 *tomatoes.* *(2)*

Question 8

Question 8

Ali conducted an investigation into the distances ridden by those members of a cycling club who rode at least 120 km in a training week. She grouped all the distances into intervals of length 10 km and then constructed a cumulative frequency diagram, shown below.



(a) Explain whether the data Ali used is a sample or a population. **[1]**

The club is taking part in a competition. Eight team members and one reserve are to be selected. The club captain decides that the team members should be those cyclists who rode the furthest during the training week, and that the reserve should be the cyclist who rode the next furthest.

(b) Use the graph to estimate the shortest distance cycled by a team member. **[1]**

The captain's best friend rode 156 km in the training week and was selected as reserve. Ali complained that this was unjustifiable.

(**a**) Explain whether the data Ali used is a sample or a population. (**1**)

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(**b**) Use the graph to estimate the shortest distance cycled by a team member. (**1**)

The captain's best friend rode 156 km in the training week and was selected as reserve. Ali complained that this was unjustifiable.

(**c**) Explain whether there is sufficient evidence in the diagram to support Ali's complaint. (**1**)

Model solution

(a) All members meeting the stated condition are included.

Population. *(1)*

(b) The eighth highest is at cumulative frequency about $60 - 8 = 52$*, which reads as approximately 162 km.*

About 162 *km.* *(1)*

(c) About 10 riders exceed 160 km, so 156 km cannot be the reserve distance after the top eight.

Yes, the complaint is justified. *(1)*

Question 9

Question 9

At the beginning of the academic year, all the pupils in year 12 at a college take part in an assessment. Summary statistics for the marks obtained by the 2021 cohort are $n = 205$, $\sum x = 23042$, $\sum x^2 = 2,591,716$. Marks may only be whole numbers, but the Head of Mathematics believes that the distribution of marks may be modelled by a Normal distribution.

(a) Calculate the mean mark and the variance of the marks. **(2)** **(b)** Use your answers to part (a) to write down a possible Normal model for the distribution of marks. **(2)** One candidate in the cohort scored less than 105. **(c)** Determine whether the model found in part (b) is consistent with this information. **(3)** **(d)** Use the model to calculate an estimate of the number of candidates who scored 115 marks. **(2)**

Model solution

(a) Use the summary-statistics formulae.

$$\bar{x} = \frac{23042}{205} = 112.4, \quad s^2 = \frac{2,591,716 - 23042^2/205}{204} = 8.800\ldots$$

Mean $= 112.4$*, variance* $= 8.8$. *(2)*

(b) Use these as the Normal mean and variance.

$X \sim N(112.4, 8.8)$. *(2)*

(c) Apply a continuity correction because marks are integers.

$$P(X < 104.5) = 0.00387\ldots, \quad 205(0.00387\ldots) = 0.794\ldots$$

One candidate is plausible, so the model is consistent. *(3)*

(d) The mark 115 corresponds to $114.5 < X < 115.5$.

$$205P(114.5 < X < 115.5) = 205(0.09155\ldots) = 18.77\ldots$$

Approximately 19 *candidates.* *(2)*

Question 10

Question 10

The parametric equations of a curve are $x = 2 + 5\cos\theta$ and $y = 1 + 5\sin\theta$, where $0 \leq \theta \leq 2\pi$. **(a)** Determine the cartesian equation of the curve. **(3)** **(b)** Hence or otherwise, find the equation of the tangent to the curve at the point $(5, -3)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be determined. **(4)**

Model solution

(a) Rearrange and use $\sin^2\theta + \cos^2\theta = 1$.

$$\left(\frac{x-2}{5}\right)^2 + \left(\frac{y-1}{5}\right)^2 = 1.$$

$(x-2)^2 + (y-1)^2 = 25$. ***(3)***

(b) The radius from $(2, 1)$ *to* $(5, -3)$ *has gradient* $-4/3$*, so the tangent has gradient* $3/4$.

$$y + 3 = \frac{3}{4}(x-5) \quad \Rightarrow \quad 3x - 4y - 27 = 0.$$

$3x - 4y - 27 = 0$. ***(4)***

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Question 11

Question 11

A die in the form of a dodecahedron has faces numbered 1 to 12. It is biased so the probability of 12 differs from every other score. The distribution of X is:

x	1	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	p	p	p	p	p	p	p	p	p	p	p	kp

where $0 < p < 1$ and k is a positive integer. Sam rolls the die 30 times, Leo 60 times and Nina 120 times; each plots a bar line graph. **(a)** Explain whose graph is most likely to give the best representation of the theoretical probability distribution. **(1)** **(b)** Find p in terms of k. **(2)** **(c)** Determine, in terms of k, the expected number of times Nina rolls a 12. **(3)** **(d)** Given that Nina rolls a 12 on 32 occasions, calculate an estimate of k. **(2)** Nina rolls the die a further 30 times. **(e)** Use your answer to part (d) to calculate an estimate for the probability that she obtains a 12 exactly 8 times in these 30 rolls. **(2)**

Model solution

(a) Relative frequencies from the largest sample are most likely to be close to theoretical probabilities.

Nina's graph. *(1)*

(b) Add all probabilities to one.

$$11p + kp = 1 \quad \Rightarrow \quad p = \frac{1}{11+k}.$$

$p = 1/(11+k)$. *(2)*

(c) Expected frequency is number of trials times probability.

$$120(kp) = \frac{120k}{11+k}.$$

$120k/(11+k)$. *(3)*

(d) Match the observed count to the expected count.

$$32 = \frac{120k}{11+k} \quad \Rightarrow \quad k = 4.$$

$k = 4$. *(2)*

(e) Then $Y \sim B(30, 4/15)$.

$$P(Y = 8) = \binom{30}{8}\left(\frac{4}{15}\right)^8\left(\frac{11}{15}\right)^{22} = 0.1628\ldots$$

0.163 *approximately.* *(2)*

Question 12

Question 12

A retailer sells bags of flour advertised as containing 1.5 kg. A trading standards officer investigates whether there is enough flour in each bag. A random sample is tested at the 5% level. Software output: sample mean 1.44, $H_0 : \mu = 1.5$, $H_1 : \mu < 1.5$, $\sigma = 0.24$, $N = 32$, SE $= 0.0424$, $Z = -1.4142$, $P = 0.0786$. (**a**) State the hypotheses the officer uses in the test, defining any parameters used. **(2)** (**b**) State the distribution used in the analysis. **(3)** (**c**) Carry out the hypothesis test, giving your conclusion in context. **(3)**

Model solution

(a) Looking for too little flour makes this a lower-tailed test.

$$H_0 : \mu = 1.5, \quad H_1 : \mu < 1.5.$$

μ is the population mean bag weight in kg. **(2)**

(b) Under the null hypothesis the sample mean has variance σ^2/n.

$$\bar{X} \sim N\left(1.5, \frac{0.24^2}{32}\right) = N(1.5, 0.0018).$$

$\bar{X} \sim N(1.5, 0.24^2/32)$. **(3)**

(c) Compare the p-value with 0.05.

$$0.0786 > 0.05.$$

Do not reject H_0. There is insufficient evidence at the 5% level that mean flour content is less than 1.5 kg. **(3)**

Question 13

Question 13

Records from the 1950s showed that 35% of human babies were born without wisdom teeth. It is believed that more babies are now born without wisdom teeth. In a random sample of 140 babies, collected in 2020, 61 were born without wisdom teeth.
The researcher stated that this shows the percentage has increased from 35%. **(a)** Explain whether this statement can be fully justified. **(1)** **(b)** Conduct a hypothesis test at the 5% level to determine whether there is evidence to suggest that more than 35% of babies are now born without wisdom teeth. **(7)**

Model solution

(a) Sampling variation means a sample result alone does not prove a population change.

No; another random sample could give a different result. *(1)*

(b) Let p be the probability that a randomly selected baby is born without wisdom teeth.

$$H_0 : p = 0.35, \quad H_1 : p > 0.35, \quad X \sim B(140, 0.35).$$

$$P(X \geq 61) = 0.02197\ldots < 0.05.$$

Reject H_0. There is sufficient evidence at the 5% level to suggest $p > 0.35$. *(7)*

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Question 14

Question 14

Fig. 14.1 shows $y = \dfrac{1}{1+x^2}$, together with 5 equal-width rectangles.

14 **Fig. 14.1** shows the curve with equation $y = \frac{1}{1+x^2}$, together with 5 rectangles of equal width.

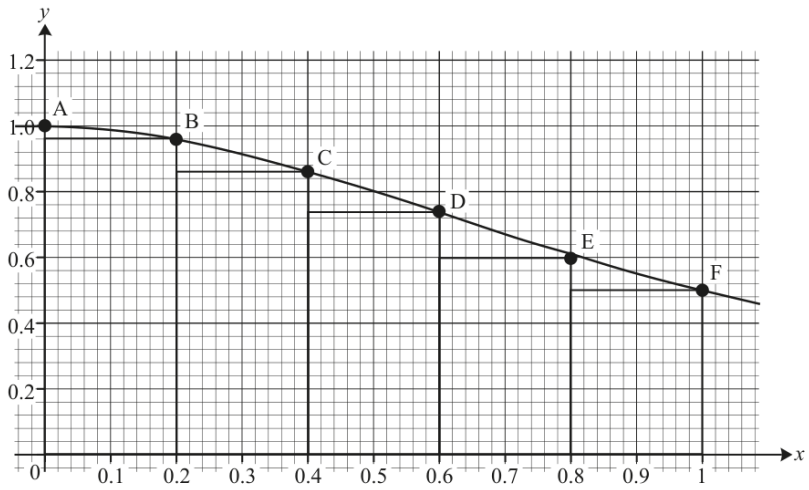


Fig. 14.1

Fig. 14.2 shows the coordinates of the points A, B, C, D, E and F.

Point	A	B	C	D	E	F
x	0	0.2	0.4	0.6	0.8	1
y	1	0.96154	0.86207	0.73529	0.60976	0.5

Fig. 14.2

(a) Use the 5 rectangles shown in **Fig. 14.1** and the information in **Fig. 14.2** to show that a lower bound for $\int_0^1 \frac{1}{1+x^2}\mathrm{d}x$ is 0.7337, correct to **4** decimal places. **[2]**

(b) Use the 5 rectangles shown in **Fig. 14.1** and the information in **Fig. 14.2** to calculate an upper bound for $\int_0^1 \frac{1}{1+x^2}\mathrm{d}x$ correct to **4** decimal places. **[2]**

(c) Hence find the length of the interval in which your answers to parts **(a)** and **(b)** indicate the

The points A–F have $x = 0, 0.2, 0.4, 0.6, 0.8, 1$ and corresponding $y = 1, 0.96154, 0.86207, 0.73529, 0.60976, 0.5$. **(a)** Show that a lower bound for $\int_0^1 \dfrac{1}{1+x^2}\,dx$ is 0.7337, correct to 4 decimal places. **(2)** **(b)** Calculate an upper bound correct to 4 decimal places. **(2)** **(c)** Hence find the interval length. **(1)**
Using $n = 10, 20, 40$, Amit obtains upper bounds $0.80998, 0.79779, 0.79162$ and lower bounds $0.75998, 0.77279, 0.77912$. **(d)** Find the smallest interval length now known. **(2)** **(e)** Without calculation, explain how a smaller interval could be found. **(1)**

Model solution

The curve decreases, so right endpoints give lower rectangles and left endpoints upper rectangles.

$$0.2(0.96154 + 0.86207 + 0.73529 + 0.60976 + 0.5) = 0.733732\ldots$$

Lower bound $= 0.7337$. *(2)*

$$0.2(1 + 0.96154 + 0.86207 + 0.73529 + 0.60976) = 0.833732\ldots$$

Upper bound $= 0.8337$. *(2)*

$0.8337 - 0.7337 = 0.1$. *(1)*

$0.79162 - 0.77912 = 0.0125$. *(2)*

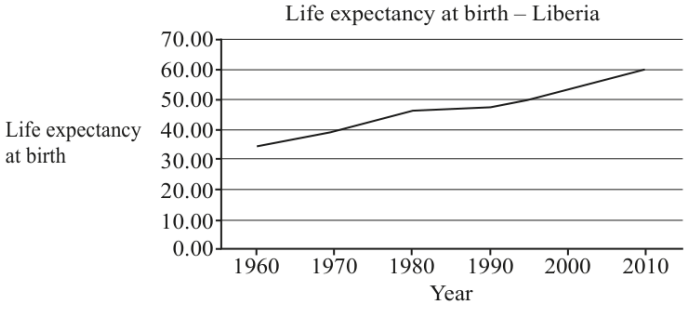
Increase n*, giving narrower rectangles.* *(1)*

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Question 15

Question 15

The pre-release material includes life expectancy at birth in countries of the world. Fig. 15.1 shows Liberia data and a time series graph.



1960	1970	1980	1990	2000	2010
34.67	39.25	46.00	47.18	52.42	59.63

Fig. 15.1

Sundip uses the LINEST function on a spreadsheet to model life expectancy as a function of calendar year by a straight line.

The equation of this line is $L = 0.473y - 892$, where L is life expectancy at birth and y is calendar year.

(a) Use this model to find an estimate of the life expectancy at birth in Liberia in 1995. **[1]**

According to the model, the life expectancy at birth in Liberia in 2025 is estimated to be 65.83 years.

(b) Explain whether each of these two estimates is likely to be reliable. **[2]**

(c) Use your knowledge of the pre-release material to explain whether this model could be used to obtain a reliable estimate of the life expectancy at birth in other countries in 1995. **[1]**

Sundip's straight-line model is $L = 0.473y - 892$, where L is life expectancy and y calendar year. (**a**) Estimate life expectancy in Liberia in 1995. (**1**) The model estimates 65.83 years in 2025. (**b**) Explain whether each estimate is likely reliable. (**2**) (**c**) Explain whether the model gives reliable estimates for other countries in 1995. (**1**) Fig. 15.2 shows Italy and South Africa.

13

Fig. 15.2 shows the life expectancy at birth between 1960 and 2010 for Italy and South Africa.

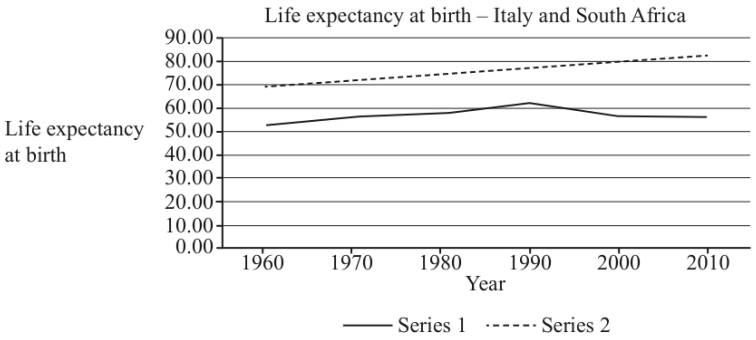


Fig. 15.2

(d) Use your knowledge of the pre-release material to

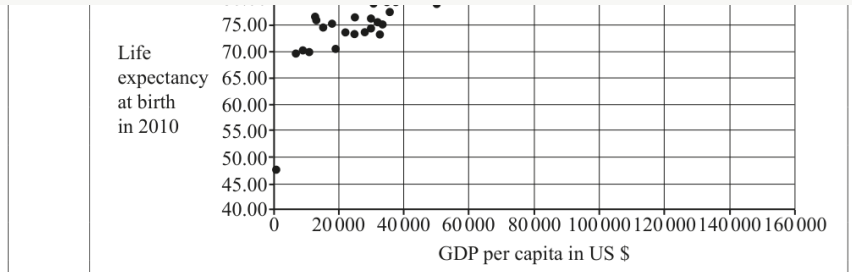
- Explain whether series 1 or series 2 represents the data for Italy.
- Explain how the data for South Africa differs from the data for most developed countries.

[2]

Turn over for question 15(e)

(**d**) Explain which series is Italy and how South Africa differs from most developed countries. (**2**)

Sundip makes the scatter diagram below for Europe but accidentally includes Central African Republic.



15(f)

(**e**) Circle its point. (**1**) Sundip states life expectancy increases as GDP per capita increases. (**f**) Explain to what extent the diagram supports this. (**2**)

Model solution

(a) Substitute 1995 in the model.

$$L = 0.473(1995) - 892 = 51.635.$$

51.64 years approximately. *(1)*

1995 is interpolation and likely reliable; 2025 is extrapolation and less reliable. *(2)*

No: life-expectancy trends vary substantially between countries. *(1)*

Series 2 is Italy (the higher European series); South Africa falls from about 1990, unlike the usual upward trend. *(2)*

Circle approximately $(700, 47.56)$. *(1)*

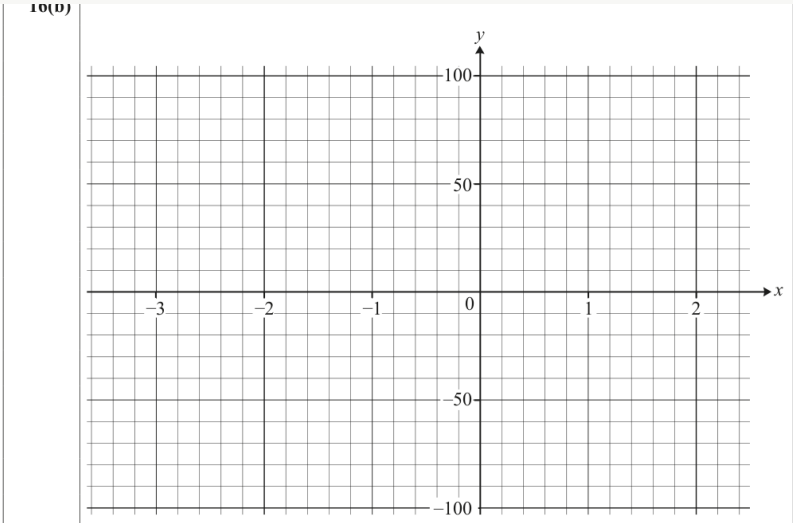
There is positive association up to about \$40,000--\$60,000; beyond this GDP range there is little association. *(2)*

Question 16

Question 16

The curve is $y = 6x^4 + 8x^3 - 21x^2 + 12x - 6$. **(a)** In this question you must show detailed reasoning. Determine the coordinates and nature of stationary points, and the x-coordinate of the non-stationary point of inflection. **(12)** **(b)** On the axes below sketch the curve.

16(b)



(3)

Model solution

(a) Stationary points have zero gradient.

$$\frac{dy}{dx} = 24x^3 + 24x^2 - 42x + 12 = 6(x+2)(2x-1)^2.$$

$$x = -2, \frac{1}{2}; \qquad y(-2) = -82, \quad y(1/2) = -\frac{31}{8}.$$

$$\frac{d^2y}{dx^2} = 72x^2 + 48x - 42 = 6(2x-1)(6x+7).$$

At $x = -2$, the second derivative is $150 > 0$, so this is a minimum. At $x = 1/2$, the gradient does

not change sign, so it is a stationary inflection. The other inflection root is $x = -7/6$.

Minimum $(-2, -82)$*; stationary inflection* $(1/2, -31/8)$*; non-stationary inflection has* $x = -7/6$. **(12)**

(b) The essential sketch features are $(0, -6)$*, roots about* -2.71 *and* 0.92*, the minimum and the stationary inflection, with both ends rising.*

Sketch the corresponding upward-opening quartic with these labelled features. **(3)**

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