Maths with Matt

Physics and maths resources

OCR A Level Mathematics B (MEI)

H640/02 - Pure Mathematics and Statistics

Tuesday 11 June 2024

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Tuesday 11 June 2024 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Calculate the exact distance between the points $(2, -1)$ and $(6, 1)$. Give your answer in the form $a\sqrt{b}$, where a and b are prime numbers. [2]

Model solution

Use Pythagoras' theorem on the horizontal and vertical changes.

$$d = \sqrt{(6-2)^2 + (1-(-1))^2} = \sqrt{16+4} = 2\sqrt{5}.$$

The exact distance is $2\sqrt{5}$. *(2)*

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Question 2

Question 2

The equation of a curve is $y = e^x$. The curve is subject to a translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ and a stretch with scale factor 2 parallel to the y-axis. Write down the equation of the new curve. [2]

Model solution

A vertical stretch multiplies every y-value by 2, and the translation then moves it down 3.

$$y = 2e^x - 3.$$

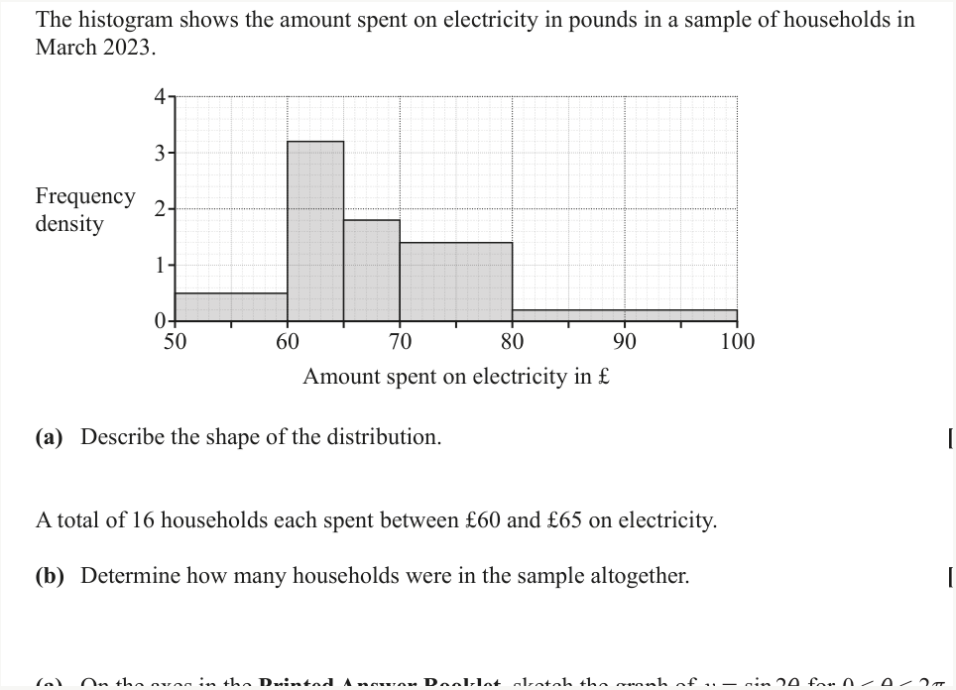
$y = 2e^x - 3.$ *(2)*

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Question 3

Question 3

The histogram shows the amount spent on electricity in pounds in a sample of households in March 2023.



(a) Describe the shape of the distribution. [1]
A total of 16 households each spent between \$60 and \$65 on electricity.
(b) Determine how many households were in the sample altogether. [2]

Model solution

(a) The longer tail is towards the larger electricity costs, so the distribution is positively skewed.

Positively skewed. *(1)*

(b) In a histogram, frequency is class width multiplied by frequency density. The stated class confirms that the vertical scale has density 3.2 *for* \$60 *to* \$65*, since* $5(3.2) = 16$*. Thus the total frequency is*

$$10(0.5) + 5(3.2) + 5(1.8) + 10(1.4) + 20(0.2) = 5 + 16 + 9 + 14 + 4 = 48.$$

There were 48 *households in the sample.* *(2)*

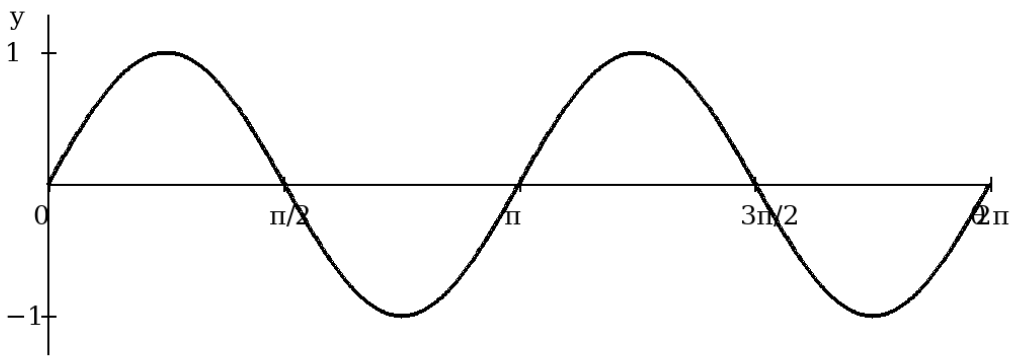
Question 4

Question 4

(a) On the axes in the Printed Answer Booklet, sketch the graph of $y = \sin 2\theta$ for $0 \leq \theta \leq 2\pi$. [2]
(b) Solve the equation $\sin 2\theta = -\frac{1}{2}$ for $0 \leq \theta \leq 2\pi$. [3]

Model solution

(a) The factor 2 gives period π, so there are two complete cycles in the stated interval; the amplitude remains 1.



A sine curve of amplitude 1, with two complete cycles from 0 to 2π, as shown. ***(2)***

(b) First solve for 2θ. The reference angle for $\sin u = -\frac{1}{2}$ is $\frac{\pi}{6}$, and sine is negative in quadrants III and IV. Since $0 \leq 2\theta \leq 4\pi$, this gives

$$2\theta = \frac{7\pi}{6}, \ \frac{11\pi}{6}, \ \frac{19\pi}{6}, \ \frac{23\pi}{6}.$$

$$\theta = \frac{7\pi}{12}, \ \frac{11\pi}{12}, \ \frac{19\pi}{12}, \ \frac{23\pi}{12}.$$

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$. ***(3)***

Question 5

Question 5

M is the event that an A-level student selected at random studies mathematics. C is the event that an A-level student selected at random studies chemistry.
You are given that $P(M) = 0.42$, $P(C) = 0.36$ and $P(M \cap C) = 0.24$. These probabilities are shown in the two-way table below.

	M	M'	Total
C	0.24		0.36
C'			
Total	0.42		1

(a) In the Printed Answer Booklet, complete the copy of the two-way table. [2]
(b) Calculate the probability that an A-level student selected at random does not study chemistry given that they do not study mathematics. [2]

Model solution

(a) Subtract the intersection from each given marginal total, then use total probability 1.

	M	M'	*Total*
C	0.24	0.12	0.36
C'	0.18	0.46	0.64
Total	0.42	0.58	1

Completed table as shown. **(2)**

(b) A conditional probability restricts the total to the M' *column.*

$$P(C' \mid M') = \frac{P(C' \cap M')}{P(M')} = \frac{0.46}{0.58} = \frac{23}{29}.$$

$\frac{23}{29}$ *(approximately* 0.793*).* **(2)**

Question 6

Question 6

The probability distribution of the discrete random variable X is shown in the table.

x	0	1	2	3
$P(X = x)$	0.2	a	$3a$	0.4

(a) Calculate the value of the constant a. [1]
(b) A single value of X is chosen at random. Find the probability that the value is an odd number. [1]
(c) Two independent values of X are chosen at random. Calculate the probability that the total of the two values is 3. [3]

Model solution

(a) The probabilities in a distribution add to 1.

$$0.2 + a + 3a + 0.4 = 1 \quad \Rightarrow \quad 4a = 0.4 \quad \Rightarrow \quad a = 0.1.$$

$a = 0.1$. ***(1)***

(b) The odd values are 1 *and* 3.

$$P(X \text{ is odd}) = P(X = 1) + P(X = 3) = 0.1 + 0.4 = 0.5.$$

0.5. ***(1)***

(c) A total of 3 *occurs as* $(0, 3), (3, 0), (1, 2)$, *or* $(2, 1)$. *Independence allows the probabilities in each ordered pair to be multiplied.*

$$2\big(0.2 \times 0.4 + 0.1 \times 0.3\big) = 2(0.11) = 0.22.$$

0.22. ***(3)***

Question 7

Question 7

A sequence is defined by the recurrence relation $u_{k+1} = u_k + 5$, with $u_1 = -2$.
(a) Write down the values of u_2, u_3, and u_4. [1]
(b) Explain whether this sequence is divergent or convergent. [1]
(c) Determine the value of u_{30}. [2]
(d) Determine the value of $\sum_{k=1}^{30} u_k$. [2]

Model solution

(a) Apply the recurrence successively, adding 5 each time.

$$u_2 = 3, \qquad u_3 = 8, \qquad u_4 = 13.$$

$u_2 = 3,\ u_3 = 8,\ u_4 = 13.$ *(1)*

(b) This is an arithmetic sequence with positive common difference 5, so its terms increase without bound rather than tending to a finite limit.

The sequence is divergent. *(1)*

(c) Use the arithmetic-sequence formula with first term -2 and common difference 5.

$$u_{30} = -2 + (30 - 1)5 = 143.$$

$u_{30} = 143.$ *(2)*

(d) The sum of an arithmetic series uses the first and last terms.

$$\sum_{k=1}^{30} u_k = \frac{30}{2}(-2 + 143) = 2115.$$

$2115.$ *(2)*

Question 8

Question 8

The equation of a curve is $y = 2x^3 + 3mx^2 - 9mx + 4$. Determine the range of values of m for which the curve has no stationary values. [6]

Model solution

Stationary values require $\frac{dy}{dx} = 0$. For there to be none, the quadratic derivative must have no real roots, so its discriminant must be negative.

$$\frac{dy}{dx} = 6x^2 + 6mx - 9m.$$

$$(6m)^2 - 4(6)(-9m) < 0$$

$$36m^2 + 216m < 0 \quad \Rightarrow \quad 36m(m+6) < 0.$$

The product is negative only between its roots -6 and 0.

$-6 < m < 0.$ **(6)**

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Question 9

Question 9

A teacher is investigating how pupils travel to and from school each day. Pupils can either travel by bus, train, car, bicycle or walk.
The teacher decides to collect a sample of size 60 for the investigation.
(a) The teacher lives in a village 10 miles away from the school. Explain how collecting a sample which just consists of pupils who live in the same village as the teacher might introduce bias. [1]
The table below shows how many students there are in each year.

Year	7	8	9	10	11
Number of students	86	105	107	101	101

(b) The teacher decides to use the method of proportional stratified sampling. Calculate the number of pupils in the sample who are in Year 9. [2]
The teacher generates a sample of 10 pupils from the 86 in Year 7 by listing them in alphabetical order and selecting the first name on the list and every ninth name thereafter.
(c) Explain whether this method will generate a simple random sample of the pupils who travel in Year 7. [1]

Model solution

(a) A village 10 miles away is unlikely to represent all travel methods: for example, it may contain no pupils who walk or cycle.

The sample would be biased towards some methods of travel and against walking or cycling. **(1)**

(b) Proportional stratification keeps each year group's sample proportion the same as in the population. The total is $86 + 105 + 107 + 101 + 101 = 500$.

$$\frac{107}{500} \times 60 = 12.84.$$

Select 13 *Year* 9 *pupils.* **(2)**

(c) A simple random sample requires every possible sample of size 10 *to have equal probability. The fixed alphabetical starting point and interval do not allow this.*

No; not every possible sample (or every pupil) has an equal probability of selection. **(1)**

Question 10

Question 10

(a) Determine the first three terms in ascending powers of x of the binomial expansion of $(8+3x)^{\frac{1}{3}}$. [4]
(b) State the range of values of x for which this expansion is valid. [1]

Model solution

(a) Factor out 8 so that the binomial theorem can be applied to an expression of the form $(1+u)^n$.

$$(8+3x)^{\frac{1}{3}} = 2\left(1+\frac{3x}{8}\right)^{\frac{1}{3}}.$$

$$\left(1+\frac{3x}{8}\right)^{\frac{1}{3}} = 1+\frac{1}{3}\left(\frac{3x}{8}\right)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}\left(\frac{3x}{8}\right)^2+\cdots = 1+\frac{x}{8}-\frac{x^2}{64}+\cdots.$$

$$(8+3x)^{\frac{1}{3}} = 2+\frac{x}{4}-\frac{x^2}{32}+\cdots.$$

$2+\frac{x}{4}-\frac{x^2}{32}$. ***(4)***

(b) An infinite binomial expansion is valid for $|u|<1$, where $u=\frac{3x}{8}$.

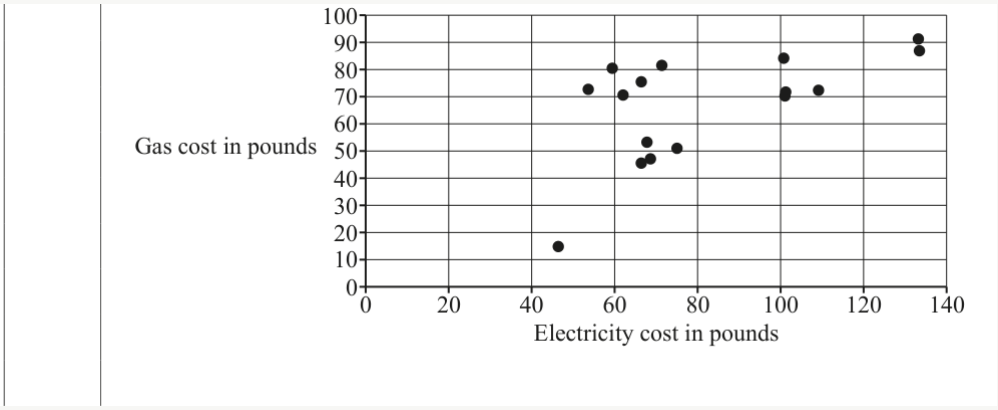
$$\left|\frac{3x}{8}\right| < 1.$$

$-\frac{8}{3} < x < \frac{8}{3}$. ***(1)***

Question 11

Question 11

A householder is investigating whether there is any relationship between his monthly cost of gas and his monthly cost of electricity, both measured in pounds ($). The householder collects a random sample of monthly costs and presents them in the scatter diagram below.



One of the points on the diagram represents the energy costs in a month when the householder was away on holiday for three weeks. The other points represent the energy costs in months when the householder did not go away on holiday.

(a) On the copy of the diagram in the Printed Answer Booklet, circle the point which represents the month when the householder was most likely to have been away on holiday for three weeks. [1]

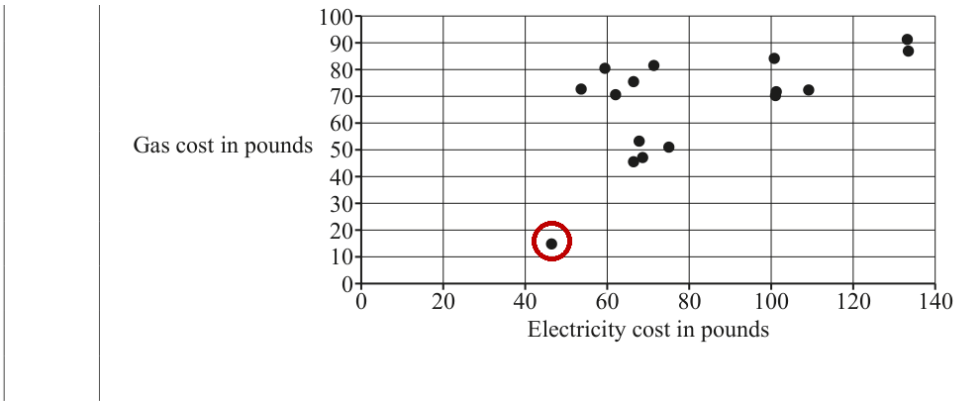
(b) With reference to the diagram, describe the relationship between the cost of gas and the cost of electricity. [1]

The householder decides to test whether there is evidence to suggest that there is any association between the monthly cost of gas and the monthly cost of electricity. The value of Spearman's rank correlation coefficient for this sample is 0.4359 and the associated p-value is $0.091\,95$.

(c) Determine whether there is any evidence to suggest, at the 5% level, that there is any association between the monthly cost of gas and the monthly cost of electricity. [3]

Model solution

(a) A three-week holiday should give unusually low costs for both utilities, so identify the isolated bottom-left point.



Circle the point approximately $(\$47, \$15)$. *(1)*

(b) The points have an upward, though rather scattered, trend.

There is a weak positive association between gas cost and electricity cost. **(1)**

(c) The given p-value is for the two-sided association test. Compare it with the 5% significance level.

$$0.09195 > 0.05.$$

There is insufficient evidence at the 5% level to suggest an association between monthly gas and electricity costs. **(3)**

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Question 12

Question 12

A survey conducted in 2021 showed that 10% of British adults were vegetarians.
A dietitian believes that the proportion of British adults who are vegetarians may have changed, so decides to conduct a hypothesis test at the 5% level of significance.
In a random sample of 112 adults, the dietitian finds that there are 19 vegetarians.
Carry out the hypothesis test to determine whether there is any evidence to support the dietitian's belief. [7]

Model solution

Let p be the probability that a British adult selected at random is vegetarian. The wording ``may have changed'' requires a two-tailed test.

$$H_0 : p = 0.10, \qquad H_1 : p \neq 0.10.$$

Under H_0, the number X of vegetarians in the sample has distribution $\mathrm{B}(112, 0.10)$. *The observed value is in the upper tail, so calculate that tail probability.*

$$P(X \geq 19) = 0.01533\ldots$$

For a two-tailed 5% test, compare one tail with 0.025.

$$0.01533 < 0.025.$$

There is sufficient evidence at the 5% level to suggest that the probability that a British adult is vegetarian is not 0.10. *(7)*

Question 13

Question 13

Determine the coordinates of the turning points on the curve with equation $y^2 + xy + x^2 - x = 1$.
[9]

Model solution

At a turning point of y against x, $\frac{dy}{dx} = 0$. Implicit differentiation is needed because y is not isolated.

$$2y\frac{dy}{dx} + y + x\frac{dy}{dx} + 2x - 1 = 0.$$

Putting $\frac{dy}{dx} = 0$ leaves a relation satisfied by any turning point:

$$y + 2x - 1 = 0, \qquad y = 1 - 2x.$$

Substitute this into the curve equation to find the corresponding x-coordinates.

$$(1 - 2x)^2 + x(1 - 2x) + x^2 - x = 1,$$

$$3x^2 - 4x = 0, \qquad x = 0 \text{ or } x = \frac{4}{3}.$$

x	0	$\frac{4}{3}$
$y = 1 - 2x$	1	$-\frac{5}{3}$

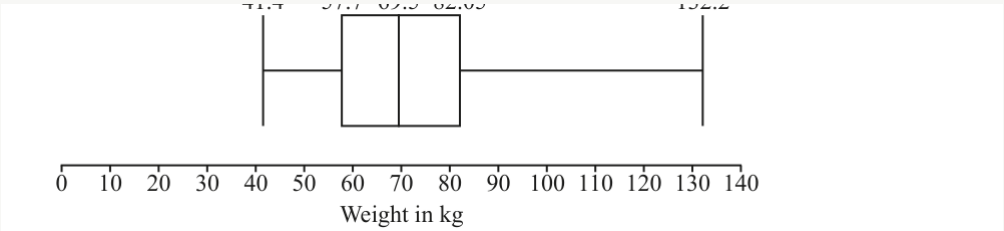
The turning points are $(0, 1)$ and $\left(\frac{4}{3}, -\frac{5}{3}\right)$. ***(9)***

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Question 14

Question 14

The pre-release material contains medical data for 103 women and 97 men.
The boxplot represents the weights in kg of 101 of the women from the pre-release material.



(a) Use your knowledge of the pre-release material to give a reason why the weights of all 103 women were **not** included in the diagram. **[1]**

(b) Determine the range of values in which any outliers lie. **[3]**

(c) Use your knowledge of the pre-release material to explain whether these outliers should be removed from any further analysis of the data. **[1]**

(a) Use your knowledge of the pre-release material to give a reason why the weights of all 103 women were not included in the diagram. [1]
(b) Determine the range of values in which any outliers lie. [3]
(c) Use your knowledge of the pre-release material to explain whether these outliers should be removed from any further analysis of the data. [1]
(d) The median weight of men in the sample was found to be 79.9 kg. Explain what may be inferred by comparing the median weight of men with the median weight of women. [1]
Further analysis of the weights of both men and women is carried out. The table shows some of the results.

	mean	standard deviation
men	82.69 kg	19.98 kg
women	72.5 kg	19.95 kg

(e) Use the information in the table to make two inferences about the distribution of the weights of men compared with the distribution of the weights of women. [2]

Model solution

(a) The pre-release data include unavailable values, so two women's weights could not be plotted.

Not all the weight data were available (two entries were missing). ***(1)***

(b) The interquartile range is $82.05 - 57.7 = 24.35$. *Use the* $1.5\,\mathrm{IQR}$ *rule.*

$$\textit{lower fence} = 57.7 - 1.5(24.35) = 21.175,$$

$$\textit{upper fence} = 82.05 + 1.5(24.35) = 118.575.$$

The plot shows no lower-tail outliers and its largest plotted value is 132.2 *kg.*

Any outliers lie in $(118.575, 132.2]$ *kg (equivalently, above* 118.575 *kg in this data set).* ***(3)***

(c) Being an outlier alone is not a reason to delete a genuine observation. The source is credible and there is no reason to think the values were misrecorded.

They should not be removed. *(1)*

(d) Medians describe a typical value. Since $79.9 > 69.5$, the typical man's weight is higher.

A typical man is heavier than a typical woman. *(1)*

(e) Compare location using the means and spread using the standard deviations.

Men's weights have a higher mean, by about 10 kg, so their distribution is located further to the right; the standard deviations are almost equal, so the dispersions are similar. *(2)*

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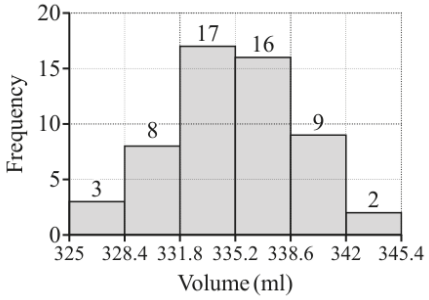
Question 15

Question 15

Bottles of Fizzipop nominally contain 330 ml of drink. A consumer affairs researcher collects a random sample of 55 bottles of Fizzipop and records the volume of drink in each bottle.
Summary statistics for the researcher's sample are shown in the table.

n	55
$\sum x$	18 535
$\sum x^2$	6 247 066.6

(a) (i) Calculate the mean volume of drink in a bottle of Fizzipop. [1]
(ii) Show that the standard deviation of the volume of drink in a bottle of Fizzipop is 3.78 ml. [1]
The researcher uses software to produce a histogram with equal class intervals, which is shown below.



(b) Explain why the researcher decides that the Normal distribution is a suitable model for the volume of drink in a bottle of Fizzipop. **[2]**

(c) Use your answers to parts **(a)** and **(b)** to determine the expected number of bottles which contain less than 330 ml in a random sample of 100 bottles. **[3]**

In order to comply with new regulations, no more than 1% of bottles of Fizzipop should contain

(b) Explain why the researcher decides that the Normal distribution is a suitable model for the volume of drink in a bottle of Fizzipop. [2]
(c) Use your answers to parts (a) and (b) to determine the expected number of bottles which contain less than 330 ml in a random sample of 100 bottles. [3]
In order to comply with new regulations, no more than 1% of bottles of Fizzipop should contain less than 330 ml.
The manufacturer decides to meet the new regulations by adjusting the manufacturing process so that the mean volume of drink in a bottle of Fizzipop is increased. The standard deviation is unaltered.
(d) Determine the minimum mean volume of drink in a bottle of Fizzipop which should ensure that the new regulations are met. Give your answer to 3 significant figures. [3]
The mean volume of drink in a bottle of Fizzipop is set to 340 ml. After several weeks the quality control manager suspects the mean volume may have reduced. She collects a random sample of 100 bottles of Fizzipop. The mean volume of drink in a bottle in the sample is found to be 339.37 ml.
(e) Assuming the standard deviation is unaltered, conduct a hypothesis test at the 5% level to determine whether there is any evidence to suggest that the mean volume of drink in a bottle of Fizzipop is less than 340 ml. [7]

Model solution

(a)(i) Divide the total volume by the number of bottles.

$$\bar{x} = \frac{18\,535}{55} = 337.$$

The mean is 337 *ml.* *(1)*

(a)(ii) Use the sample standard-deviation formula because the figures are from a sample.

$$s = \sqrt{\frac{1}{55-1}\left(6\,247\,066.6 - 55(337)^2\right)} = \sqrt{14.288\ldots} = 3.78 \text{ ml}.$$

$s = 3.78$ *ml, as required.* *(1)*

(b) The histogram is approximately symmetrical and bell-shaped, with one peak; volume is also continuous. These are suitable features for a Normal model.

The distribution is approximately symmetrical and bell-shaped (and is continuous), so a Normal model is suitable. *(2)*

(c) Model an individual bottle as $X \sim \mathrm{N}(337, 3.78^2)$*. Standardising the required cut-off gives*

$$P(X < 330) = P\left(Z < \frac{330 - 337}{3.78}\right) = P(Z < -1.852\ldots) = 0.032\ldots.$$

$$100(0.032\ldots) = 3.2\ldots.$$

The expected number is approximately 3.2 *bottles.* *(3)*

(d) Set the lower-tail probability equal to its largest permitted value, 0.01*. The associated standard Normal value is* -2.3263*.*

$$\frac{330 - \mu}{3.78} = -2.3263,$$

$$\mu = 330 + 2.3263(3.78) = 338.79\ldots.$$

The minimum mean is 339 *ml (to* 3 *significant figures).* *(3)*

(e) Let μ *be the population mean bottle volume. The manager suspects a reduction, so this is a lower-tailed test.*

$$H_0 : \mu = 340, \qquad H_1 : \mu < 340.$$

Under H_0*, the sample mean has distribution*

$$\bar{X} \sim \mathrm{N}\left(340, \frac{3.78^2}{100}\right).$$

$$P(\bar{X} < 339.37) = P\left(Z < \frac{339.37 - 340}{3.78/10}\right) = P(Z < -1.667\ldots) = 0.0477\ldots.$$

This is below 0.05*, so the observed mean is sufficiently unusual if the mean has remained* 340 *ml.*

There is sufficient evidence at the 5% *level to suggest that the mean volume of drink in a bottle of Fizzipop is less than* 340 *ml.* *(7)*

Question 16

Question 16

In this question you must show detailed reasoning.
Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{9y}{(x-1)(x+2)},$$

given that $x = 2$ when $y = 16$. [12]

Model solution

The right-hand side is a product of a function of x and y, so separate variables before integrating.

$$\frac{1}{y}\,dy = \frac{9}{(x-1)(x+2)}\,dx.$$

Partial fractions make the rational function straightforward to integrate.

$$\frac{9}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

$$9 = A(x+2) + B(x-1).$$

Putting $x = 1$ gives $A = 3$, and putting $x = -2$ gives $B = -3$. Hence

$$\int \frac{1}{y}\,dy = \int \left(\frac{3}{x-1} - \frac{3}{x+2}\right) dx,$$

$$\ln y = 3\ln(x-1) - 3\ln(x+2) + c.$$

Use the given point to determine the integration constant.

$$\ln 16 = 3\ln(2-1) - 3\ln(2+2) + c = -3\ln 4 + c,$$

$$c = \ln 16 + 3\ln 4 = 5\ln 4 = \ln 1024.$$

Combine the logarithms and exponentiate to make y the subject.

$$\ln y = \ln\left(\frac{1024(x-1)^3}{(x+2)^3}\right).$$

$$y = \frac{1024(x-1)^3}{(x+2)^3}.$$ ***(12)***