Maths with Matt

Physics and maths resources

OCR A Level Mathematics B (MEI)

H640/02 - Pure Mathematics and Statistics

Wednesday 13 October 2021

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Wednesday 13 October 2021 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

The equation of a curve is $y = 4x^2 + 8x + 1$. The curve is stretched parallel to the x-axis with scale factor 2. Find the equation of the new curve, giving your answer in the form $y = ax^2 + bx + c$, where a, b and c are integers to be determined. **(2)**

Model solution

A stretch parallel to the x-axis replaces x by $x/2$, since every horizontal coordinate is doubled.

$$y = 4\left(\frac{x}{2}\right)^2 + 8\left(\frac{x}{2}\right) + 1 = x^2 + 4x + 1.$$

$y = x^2 + 4x + 1.$ *(2)*

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Question 2

Question 2

(a) Write $65°$ in radians, giving your answer in the form $k\pi$, where k is a fraction in its lowest terms. **(2)**

(b) Write 0.211 radians in degrees, giving your answer correct to 1 decimal place. **(1)**

Model solution

(a) Convert degrees to radians by multiplying by $\pi/180$.

$$65 \times \frac{\pi}{180} = \frac{13\pi}{36}.$$

(a) $\dfrac{13\pi}{36}$. *(2)*

(b) The reverse conversion multiplies radians by $180/\pi$.

$$0.211 \times \frac{180}{\pi} = 12.089\ldots$$

(b) $12.1°$. *(1)*

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Question 3

Question 3

Draw a number line to show the values of x which belong to the set $\{x : x \geq 2\} \cap \{x : x < 7\}$. **(3)**

Model solution

The intersection contains values satisfying both conditions: include 2, but exclude 7.



$2 \leq x < 7$. *(3)*

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Question 4

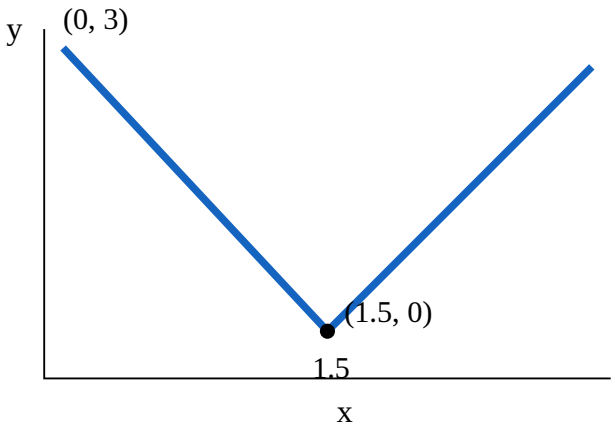
Question 4

Sketch the graph of $y = |2x - 3|$. **(3)**

Model solution

An absolute-value graph is made from the line $y = 2x - 3$ by reflecting every part below the x-axis upwards. The zero of the line gives the vertex.

$$2x - 3 = 0 \quad \Rightarrow \quad x = 1.5, \qquad y(0) = 3.$$



A V-shaped graph with vertex $(1.5, 0)$, symmetric about $x = 1.5$, passing through $(0, 3)$.
(3)

Question 5

Question 5

It is known that 40% of people in Britain carry a certain gene. A random sample of 32 people is collected.
(a) Calculate the probability that exactly 12 people carry the gene. **(1)**
(b) Calculate the probability that at least 8 people carry the gene, giving your answer correct to 3 decimal places. **(2)**

Model solution

Each person has two outcomes and the probability is assumed constant, so X, the number carrying the gene, has $X \sim \text{B}(32, 0.4)$.

$$P(X = 12) = \binom{32}{12}(0.4)^{12}(0.6)^{20} = 0.1385\ldots$$

(a) 0.139 *(or* 0.14*).* *(1)*

(b) A complement is quickest because the unwanted values are only 0 *to* 7*.*

$$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.0248219\ldots = 0.975178\ldots$$

(b) 0.975*.* *(2)*

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Question 6

Question 6

You are given that $\mathbf{v} = 2\mathbf{a} + 3\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are the position vectors $\mathbf{a} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$.

(a) Determine the magnitude of $\mathbf{v}$. **(3)**

(b) Determine the angle between $\mathbf{v}$ and the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. **(2)**

Model solution

(a) First form the vector component by component, then use Pythagoras for its magnitude.

$$\mathbf{v} = 2\begin{pmatrix} 5 \\ 3 \end{pmatrix} + 3\begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 \\ 24 \end{pmatrix}, \qquad |\mathbf{v}| = \sqrt{7^2 + 24^2} = 25.$$

(a) 25. *(3)*

(b) The requested angle is measured from the positive x-direction, so use the component ratio $\tan\theta = 24/7$.

$$\theta = \tan^{-1}\left(\frac{24}{7}\right) = 73.74\ldots^\circ.$$

(b) 73.7° *(approximately).* *(2)*

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Question 7

Question 7

The parametric equations of a circle are $x = 7 + 5\cos\theta$, $y = 5\sin\theta - 3$, for $0 \leq \theta \leq 2\pi$.
(a) Find a cartesian equation of the circle. **(3)**
(b) State the coordinates of the centre of the circle. **(1)**

Model solution

(a) Rearranging each equation isolates the sine and cosine. Squaring and adding removes the parameter using $\cos^2\theta + \sin^2\theta = 1$.

$$x - 7 = 5\cos\theta, \qquad y + 3 = 5\sin\theta,$$

$$(x-7)^2 + (y+3)^2 = 25(\cos^2\theta + \sin^2\theta) = 25.$$

(a) $(x-7)^2 + (y+3)^2 = 25$. *(3)*

(b) Compare with $(x-a)^2 + (y-b)^2 = r^2$.

(b) $(7, -3)$. *(1)*

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Question 8

Question 8

The Normal variable X is transformed to the Normal variable Y. The transformation is $y = a + bx$, where a and b are positive constants. You are given that $X \sim \mathrm{N}(42, 6.8)$ and $Y \sim \mathrm{N}(57.2, 11.492)$. Determine the values of a and b. **(4)**

Model solution

For $Y = a + bX$, the mean changes to $a + b\mu_X$, while the variance is multiplied by b^2. Use the variances first because they give b directly.

$$b^2(6.8) = 11.492 \quad \Rightarrow \quad b^2 = 1.69 \quad \Rightarrow \quad b = 1.3,$$

$$a + 1.3(42) = 57.2 \quad \Rightarrow \quad a = 2.6.$$

$a = 2.6$, $b = 1.3$. ***(4)***

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Question 9

Question 9

Labrador puppies may be black, yellow or chocolate in colour. Some information about a litter of 9 puppies is given in the table.

	male	female
black	1	3
yellow	2	1
chocolate	1	1

Four puppies are chosen at random to train as guide dogs.

(a) Determine the probability that exactly 3 females are chosen. **(3)**

(b) Determine the probability that at least 3 black puppies are chosen. **(3)**

(c) Determine the probability that exactly 3 females are chosen given that at least 3 black puppies are chosen. **(3)**

(d) Explain whether the two events 'choosing exactly 3 females' and 'choosing at least 3 black puppies' are independent events. **(1)**

Model solution

There are 5 females, 4 males and 4 black puppies. Selection is without replacement, so combinations count equally likely groups.

$$P(F) = \frac{\binom{5}{3}\binom{4}{1}}{\binom{9}{4}} = \frac{40}{126} = \frac{20}{63}.$$

(a) $\frac{20}{63}$. *(3)*

(b) 'At least 3' means either all four black puppies, or three black and one non-black.

$$P(B) = \frac{\binom{4}{4}\binom{5}{0} + \binom{4}{3}\binom{5}{1}}{\binom{9}{4}} = \frac{1+20}{126} = \frac{1}{6}.$$

(b) $\frac{1}{6}$. *(3)*

(c) Count the selections meeting both conditions. They consist of either 3 black females and one male, or 2 black females, one non-black female and the black male.

$$P(F \cap B) = \frac{\binom{3}{3}\binom{4}{1} + \binom{3}{2}\binom{2}{1}\binom{1}{1}}{\binom{9}{4}} = \frac{10}{126} = \frac{5}{63},$$

$$P(F \mid B) = \frac{P(F \cap B)}{P(B)} = \frac{5/63}{1/6} = \frac{10}{21}.$$

(c) $\frac{10}{21}$. *(3)*

(d) Independent events would have $P(F \mid B) = P(F)$. These values differ.

(d) No: $\frac{10}{21} \neq \frac{20}{63}$, *so the events are not independent.* *(1)*

Question 10

Question 10

Ben has an interest in birdwatching. For many years he has identified, at the start of the year, 32 days on which he will spend an hour counting the number of birds he sees in his garden. He divides the year into four using the Meteorological Office definition of seasons. Each year he uses stratified sampling to identify the 32 days on which he will count the birds in his garden, drawn equally from the four seasons.
Ben's data for 2019 are shown in the stem and leaf diagram in Fig. 10.1.

0	3	5	9	9	9						
1	0	0	1	1	2	4	5	6	7	8	9
2	0	1	4	6	7	8	9				
3	0	0	2	3							
4	0	3	6								
5	1										
6	0										

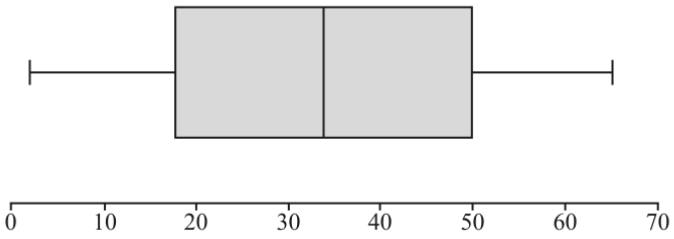
Fig. 10.1

(a) Suggest a reason why Ben chose to use stratified sampling instead of simple random sampling. **(1)**
(b) Describe the shape of the distribution. **(1)**
(c) Explain why the mode is not a useful measure of central tendency in this case. **(1)**
(d) For Ben's sample, determine the median and the interquartile range. **(3)**
Ben found a boxplot for the sample of size 32 he collected using stratified sampling in 2015. The boxplot is shown in Fig. 10.2.

- the median,
- the interquartile range. [3

Ben found a boxplot for the sample of size 32 he collected using stratified sampling in 2015.

The boxplot is shown in **Fig. 10.2**.



(e) With reference to Fig. 10.2 and your answer to part (d), comment on whether there is any evidence to support Ben's belief that he sees fewer birds in his garden, and that the number of birds he sees is more variable. **(2)**
Jane says she can tell that the data for 2015 is definitely uniformly distributed by looking at the boxplot.
(f) Explain why Jane is wrong. **(1)**

Model solution

(a) The seasons are natural strata. Sampling each one makes sure seasonal differences in bird numbers are represented.

(a) It accounts for seasonal variation in the birds seen. *(1)*

(b) Most observations are low, with a longer tail towards large values.

(b) Positively skewed. *(1)*

(c) The mode is 9, but it is in the lower tail and is not representative of the centre of this skewed distribution.

(c) The mode is not a useful central value here because the distribution is skewed. *(1)*

(d) There are 32 values. The median is the mean of the 16th and 17th values; quartiles are found from the centres of the lower and upper halves.

$$\textit{median} = \frac{18+21}{2} = 19.5, \qquad Q_1 = 11, \quad Q_3 = 30, \qquad \text{IQR} = 30 - 11 = 19.$$

(d) Median $= 19.5$; interquartile range $= 19$. *(3)*

(e) The 2015 median is about 33, much larger than 19.5, so the 2019 data support the claim of fewer birds. Its IQR is about 32, larger than 19, so the data do not support the claim that variability has increased.

(e) Evidence supports fewer birds, but not greater variability. *(2)*

(f) A boxplot only gives summary positions; it does not show how observations are spread within each quartile. Different distributions can produce the same boxplot.

(f) A boxplot cannot establish that the distribution is uniform. *(1)*

Question 11

Question 11

In 2010 the heights of adult women in the UK were found to have mean $\mu = 161.6$ cm and variance $\sigma^2 = 1.96$ cm^2. It is believed that the mean height of adult women in 2020 in the UK is greater than in 2010. In 2020 a researcher collected a random sample of the heights of 200 adult women in the UK. The researcher calculated the sample mean height and carried out a hypothesis test at the 5% level to investigate whether there was any evidence to suggest that the mean height of adult women in the UK had increased. The researcher assumed that the variance was unaltered.

(a) State suitable hypotheses for the test, defining any variables you use. Explain whether the researcher conducted a 1-tail or a 2-tail test. **(3)**

(b) Determine the critical region for the test. **(2)**

The researcher found that the sample mean was 161.9 cm and made the following statements.

- The sample mean is in the critical region.
- The null hypothesis is accepted.
- This proves that the mean height of adult women in the UK is unaltered at 161.6 cm.

(c) Explain whether each of these statements is correct. **(3)**

Model solution

(a) Let μ be the population mean height of adult women in the UK in 2020. The word 'greater' determines a one-tailed, upper-tail test.

$$H_0 : \mu = 161.6, \qquad H_1 : \mu > 161.6.$$

(a) The hypotheses above; this is a one-tailed upper-tail test because an increase is being tested. *(3)*

(b) Under H_0, $\overline{X} \sim \mathrm{N}\left(161.6, \frac{1.96}{200}\right)$. The upper 5% critical value has $z = 1.645$.

$$\overline{x} = 161.6 + 1.645\sqrt{\frac{1.96}{200}} = 161.7628\ldots$$

(b) Critical region $\overline{X} > 161.763\ldots$, or $\overline{X} \geq 161.8$ cm. *(2)*

(c) Since $161.9 > 161.8$, the first statement is correct. A value in the critical region means reject H_0, so the second is incorrect. A hypothesis test provides evidence rather than proof, so the third is also incorrect.

(c) First correct; second incorrect; third incorrect. *(3)*

Question 12

Question 12

Fig. 12.1 shows an excerpt from the pre-release material.

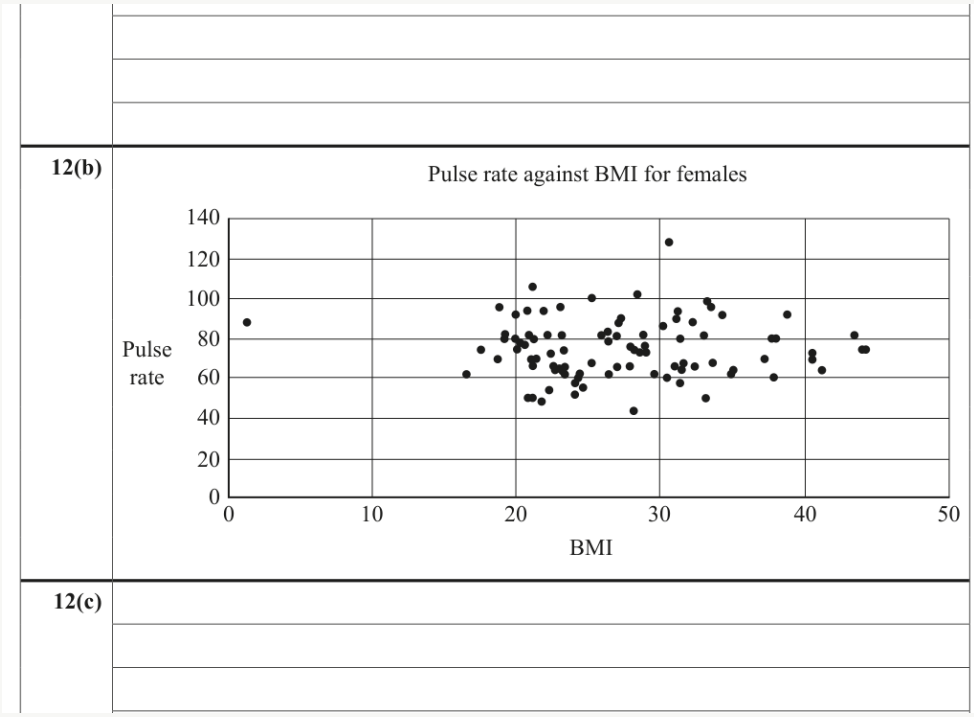
	A	B	C	D	E	F	G	H
1	Sex	Age	Marital	Weight	Height	BMI	Waist	Pulse
2	Female	34	Married	60.3	173.4	20.05	82.5	74
3	Female	85	Widowed	64.7	161.2	24.9	#N/A	#N/A
4	Female	48	Divorced	100.6	171.4	34.24	105.6	92
5	Male	61	Married	70.9	169.5	24.68	92.2	70
6	Male	68	Divorced	96.8	181.6	29.35	112.9	68

Fig. 12.1

There was no data available for cell H3.

(a) Explain why #N/A is used when no data is available. **(1)**

Fig. 12.2 shows a scatter diagram of pulse rate against BMI (Body Mass Index) for females. All the available data was used.



There are two outliers on the diagram.

(b) On the copy of Fig. 12.2 in the Printed Answer Booklet, ring these outliers. **(1)**

(c) Use your knowledge of the pre-release material to explain whether either of these outliers should be removed. **(2)**

(d) State whether the diagram suggests there is any correlation between pulse rate and BMI. **(1)**

The product moment correlation coefficient between waist measurement, w, in cm and BMI, b, for females was found to be 0.912. All the available data was used.

(e) Explain why a model of the form $w = mb + c$ for the relationship between waist measurement and BMI is likely to be appropriate. **(1)**

The LINEST function on a spreadsheet gives $m = 2.16$ and $c = 33.0$.

(f) Calculate an estimate of the value for cell G3 in Fig. 12.1. **(1)**

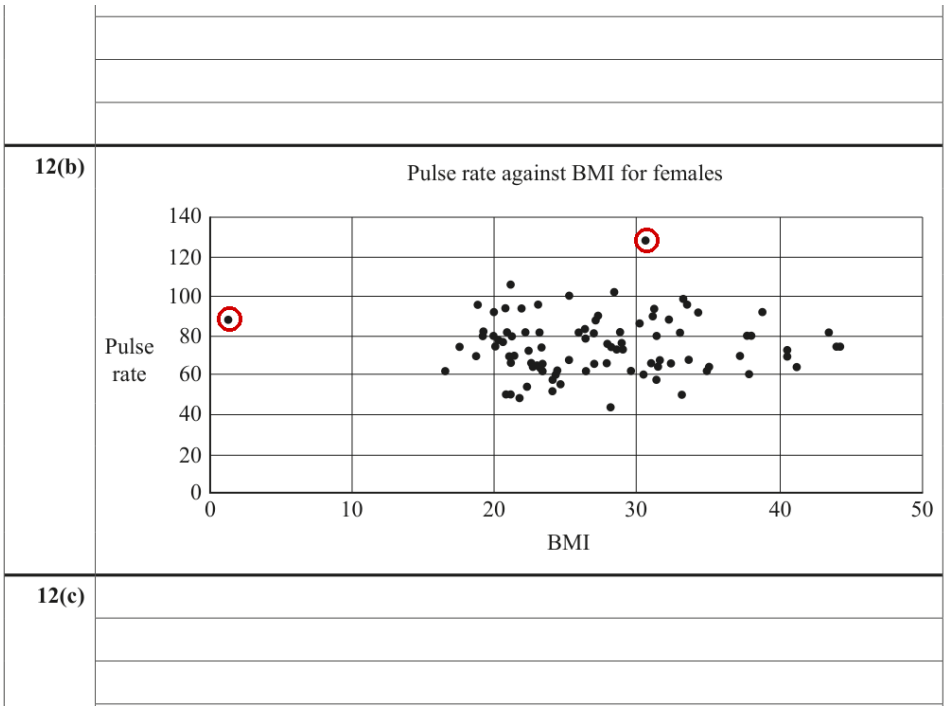
Model solution

(a) Missing data should not be treated as a genuine value of zero.

(a) $\#\mathrm{N/A}$ *makes the software ignore the cell rather than read it as zero.* *(1)*

(b) The isolated points are the point with extremely low BMI and the point with very high pulse rate.

(b) Ring $(1.3, 88)$ *and* $(30.6, 128)$. *(1)*



(c) A BMI of 1.3 *is impossible for a person, so it is an error and should be removed. A pulse rate of* 128 *is high but possible, so there is no reason to remove it.*

(c) Remove $(1.3, 88)$*; retain* $(30.6, 128)$. *(2)*

(d) The points show no clear upward or downward trend.

(d) No correlation is suggested. *(1)*

(e) The PMCC 0.912 *is close to* 1*, indicating strong positive linear correlation.*

(e) A linear model is appropriate because there is strong positive correlation. *(1)*

(f) Substitute the BMI in row 3 into the regression model.

$$w = 2.16(24.9) + 33.0 = 86.784.$$

(f) 86.784 *cm, approximately* 86.8 *cm.* *(1)*

Question 13

Question 13

At a certain factory Christmas tree decorations are packed in boxes of 10. The quality control manager collects a random sample of 100 boxes of decorations and records the number of decorations in each box which are damaged. His results are displayed in Fig. 13.1.

Number of damaged decorations	0	1	2	3	4	5 or more
Number of boxes	19	35	28	13	5	0

Fig. 13.1

(a) Calculate the mean number of damaged decorations per box and the standard deviation of the number of damaged decorations per box. **(2)**

It is believed that the number of damaged decorations in a box of 10, X, may be modelled by a binomial distribution such that $X \sim \text{B}(n, p)$.

(b) State suitable values for n and p. **(1)**

(c) Use the binomial model to complete the copy of Fig. 13.2 in the Printed Answer Booklet, giving your answers correct to 1 decimal place.

Number of damaged decorations	0	1	2	3	4	5 or more
Observed number of boxes	19	35	28	13	5	0
Expected number of boxes						

Fig. 13.2

(3)

(d) Explain whether the model is a good fit for these data. **(1)**

Model solution

(a) Use the frequency table. Since there are no boxes with 5 or more damaged decorations, the listed values give the complete calculation.

$$\bar{x} = \frac{0(19) + 1(35) + 2(28) + 3(13) + 4(5)}{100} = 1.5,$$

$$s = \sqrt{\frac{0^2(19) + 1^2(35) + 2^2(28) + 3^2(13) + 4^2(5)}{100} - 1.5^2} = 1.095\ldots$$

(a) Mean $= 1.5$; standard deviation ≈ 1.10. *(2)*

(b) Each box contains 10 decorations. Estimate the probability of damage by the sample mean number damaged per box divided by 10.

$$p = \frac{1.5}{10} = 0.15.$$

(b) $n = 10$, $p = 0.15$. *(1)*

(c) With $X \sim \text{B}(10, 0.15)$, multiply each probability by 100 boxes. The last category is found by complement.

x	0	1	2	3	4	5 *or more*
$100P(X = x)$	19.7	34.7	27.6	13.0	4.0	1.0

(c) Expected frequencies: 19.7, 34.7, 27.6, 13.0, 4.0, 1.0. *(3)*

(d) The expected frequencies are close to the observed frequencies across the categories.

(d) Yes, the binomial model is a good fit. *(1)*

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Question 14

Question 14

The equation of a curve is $y = x^2(x-2)^3$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in factorised form. **(4)**

(b) Determine the coordinates of the stationary points on the curve. **(4)**

In part (c) you may use the result $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4(x-2)(5x^2-8x+2)$.

(c) Determine the nature of the stationary points on the curve. **(3)**

(d) Sketch the curve. **(2)**

Model solution

(a) This is a product, so use the product rule. The chain rule differentiates $(x-2)^3$. Then factor the result to make the stationary points clear.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x(x-2)^3 + x^2\big(3(x-2)^2\big)$$

$$= x(x-2)^2\big(2(x-2)+3x\big) = x(x-2)^2(5x-4).$$

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x(x-2)^2(5x-4)$. *(4)*

(b) Stationary points occur where the derivative is zero. Substitute each resulting x-coordinate into the original equation.

$$x(x-2)^2(5x-4) = 0 \quad \Rightarrow \quad x = 0,\ 2,\ \frac{4}{5}.$$

$$y(0) = 0, \qquad y(2) = 0, \qquad y\left(\frac{4}{5}\right) = \left(\frac{4}{5}\right)^2\left(-\frac{6}{5}\right)^3 = -\frac{3456}{3125}.$$

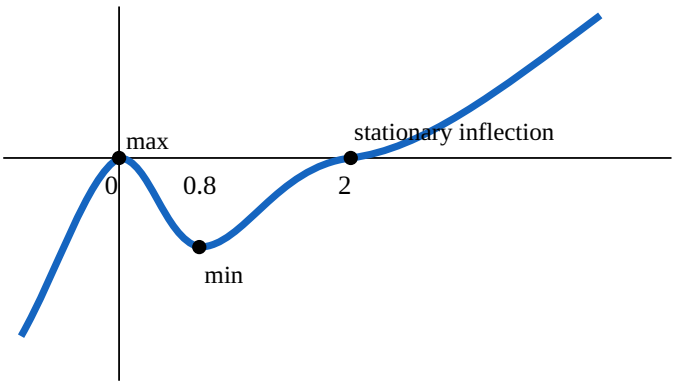
(b) $(0,0)$, $\left(\dfrac{4}{5}, -\dfrac{3456}{3125}\right)$, $(2,0)$. *(4)*

(c) Apply the second-derivative test where it is decisive. At $x = 2$, the repeated derivative factor means the gradient does not change sign; checking either side identifies a stationary point of inflection.

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}(0) = -16 < 0, \qquad \frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(\frac{4}{5}\right) = 5.76 > 0.$$

(c) $(0,0)$ *is a local maximum;* $\left(\dfrac{4}{5}, -\dfrac{3456}{3125}\right)$ *is a local minimum;* $(2,0)$ *is a stationary point of inflection.* *(3)*

(d) The curve has the stationary points found above; it also crosses the x-axis at $x = 0$ and has a flattened crossing at $x = 2$.



Sketch shown, with the maximum, minimum and stationary point of inflection identified. *(2)*

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Question 15

Question 15

(a) Show that $0.01\sum_{r=1}^{\infty}0.99^{r-1}=1$. **(3)**

Kofi is a very good table tennis player. Layla is determined to beat him. Every week they play one match of table tennis against each other. They will stop playing when Layla wins the match for the first time. X is the discrete random variable 'the number of matches they play in total'.

Kofi models the situation using the probability function $P(X=r)=0.99^{r-1}\times0.01, r=1,2,3,4,\ldots$.

Kofi states that he is 95% certain that Layla will not beat him within 6 years.

(b) Determine whether Kofi's statement is consistent with his model. **(3)**

In between matches, Layla practises, but Kofi does not.

(c) Explain why Layla might disagree with Kofi's model. **(1)**

Layla models the situation using the probability function $P(X=r)=kr^2, r=1,2,3,4,5,6,7,8$.

(d) Explain how Layla's model takes into account the fact that she practises between matches, but Kofi's does not. **(1)**

Layla states that she is 95% certain that she will beat Kofi within the first 6 matches.

(e) Determine whether Layla's statement is consistent with her model. **(3)**

Model solution

(a) This is an infinite geometric series with first term $a=0.01$ and common ratio $r=0.99$. Since $|r|<1$, its sum is $a/(1-r)$.

$$0.01\sum_{r=1}^{\infty}0.99^{r-1}=\frac{0.01}{1-0.99}=1.$$

(a) The probabilities sum to 1. ***(3)***

(b) Six years contains $6\times52=312$ weekly matches. The model gives the probability that Layla wins within this time.

$$P(X\leq312)=1-0.99^{312}=0.9565\ldots$$

This is greater than 0.95, so the model says Layla is more than 95% likely to beat him within six years, not avoid beating him.

(b) No. The statement is not consistent with the model. ***(3)***

(c) Kofi's geometric model assumes the same 0.01 probability of Layla winning every match.

(c) Layla may improve through practice, so her chance of winning should increase over time. ***(1)***

(d) In Layla's model, kr^2 increases as r increases.

(d) It makes Layla's probability of winning increase from match to match. ***(1)***

(e) First normalise the probability function, then add the probabilities for matches 1 to 6.

$$k(1+4+9+16+25+36+49+64)=1,\qquad k=\frac{1}{204},$$

$$P(X\leq6)=\frac{1+4+9+16+25+36}{204}=\frac{91}{204}=0.446\ldots$$

(e) No: $0.446\ldots < 0.95$*, so her statement is not consistent with the model.* *(3)*

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Question 16

Question 16

In this question you must show detailed reasoning. Find $\int \frac{\sqrt{x}}{x+1}\,\mathrm{d}x$. **(8)**

Model solution

Use the substitution $x = u^2$ because it removes the square root and turns the integrand into a rational function of u.

$$x = u^2, \qquad \sqrt{x} = u, \qquad \mathrm{d}x = 2u\,\mathrm{d}u.$$

$$\int \frac{\sqrt{x}}{x+1}\,\mathrm{d}x = \int \frac{u}{u^2+1}(2u)\,\mathrm{d}u = \int \frac{2u^2}{u^2+1}\,\mathrm{d}u.$$

Divide the rational expression before integrating.

$$\frac{2u^2}{u^2+1} = 2 - \frac{2}{u^2+1},$$

$$\int \left(2 - \frac{2}{u^2+1}\right)\mathrm{d}u = 2u - 2\tan^{-1} u + C.$$

Finally substitute $u = \sqrt{x}$ back into the antiderivative.

$2\sqrt{x} - 2\tan^{-1}(\sqrt{x}) + C.$ ***(8)***

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