Maths with Matt

Physics and maths resources

OCR A Level Mathematics B (MEI)

H640/03 - Pure Mathematics and Comprehension

Thursday 19 June 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Thursday 19 June 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Question wording is reproduced from the user-provided 2025 paper for study/reference. OCR retains copyright in the original examination material.

Contents

Question 1 2

Question 2 3

Question 3 4

Question 4 6

Question 5 7

Question 6 10

Question 7 11

Question 8 13

Question 9 14

Question 10 15

Question 11 16

Question 12 17

Question 13 18

Question 14 19

www.mathswithmatt.co.uk

Question 1

Question 1

(a) Find the first three terms in the binomial expansion of $(1+3x)^{1/2}$. [3]
(b) State the range of values of x for which this expansion is valid. [1]

Model solution

(a) The exponent is not a positive integer, so use the general binomial expansion and retain the constant, linear and quadratic terms.

$$(1+3x)^{1/2} = 1 + \frac{1}{2}(3x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(3x)^2 + \cdots = 1 + \frac{3x}{2} - \frac{9x^2}{8} + \cdots$$

$1 + \dfrac{3x}{2} - \dfrac{9x^2}{8}$. *(3)*

(b) For this binomial expansion the quantity added to 1*, namely* $3x$*, must have modulus less than* 1*.*

$$|3x| < 1 \quad \Longrightarrow \quad |x| < \frac{1}{3}.$$

$-\dfrac{1}{3} < x < \dfrac{1}{3}$. *(1)*

www.mathswithmatt.co.uk

Question 2

Question 2

You are given that $f(x) = \dfrac{3}{x}$ on the domain $\{x : x \in \mathbb{R},\ x \neq 0\}$, and $g(x) = x - 1$ on the domain $\{x : x \in \mathbb{R}\}$. Find $gf(x)$. You must state the domain. [2]

Model solution

The notation $gf(x)$ means $g(f(x))$: first apply f, then subtract 1. The restriction from f remains.

$$gf(x) = g\left(\frac{3}{x}\right) = \frac{3}{x} - 1 = \frac{3 - x}{x}.$$

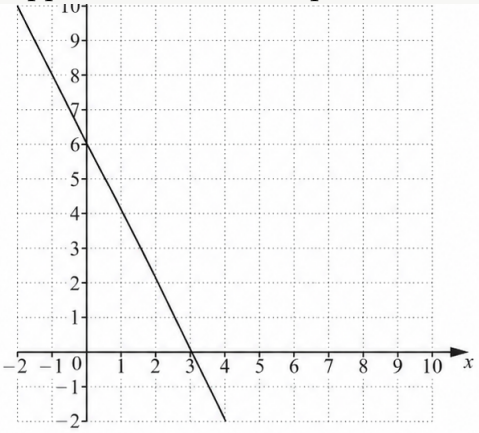
$gf(x) = \dfrac{3}{x} - 1$, *with domain* $\{x : x \in \mathbb{R},\ x \neq 0\}$. *(2)*

www.mathswithmatt.co.uk

Question 3

Question 3

The straight line with equation $2x + y = 6$ crosses the x-axis at A and the y-axis at B.
(a) Draw the line with equation $2x + y = 6$ on the grid in the Printed Answer Booklet. [1]
(b) Determine the equation of the perpendicular bisector of AB. [6]
(c) Points A and B are opposite vertices of a square of side a. Determine the exact value of a. [3]



3(b)

Model solution

(a) Intercepts are the quickest way to plot a line. Setting one coordinate to zero gives two points on it.

$$y = 0 \Rightarrow x = 3, \qquad x = 0 \Rightarrow y = 6.$$

The straight line through $(3, 0)$ and $(0, 6)$ is plotted on the grid. **(1)**

(b) A perpendicular bisector passes through the midpoint of AB, and its gradient is the negative reciprocal of the gradient of AB.

$$A = (3, 0), \quad B = (0, 6), \quad M = \left(\frac{3+0}{2}, \frac{0+6}{2}\right) = \left(\frac{3}{2}, 3\right),$$

$$m_{AB} = \frac{6-0}{0-3} = -2, \qquad m_{\perp} = \frac{1}{2}.$$

$$y - 3 = \frac{1}{2}\left(x - \frac{3}{2}\right) \quad \Longrightarrow \quad y = \frac{1}{2}x + \frac{9}{4}.$$

$y = \frac{1}{2}x + \frac{9}{4}$. **(6)**

(c) Since A and B are opposite vertices, AB is the diagonal. A square with side a has diagonal satisfying $AB^2 = a^2 + a^2$.

$$AB^2 = (3-0)^2 + (0-6)^2 = 45, \qquad 2a^2 = 45, \qquad a = \sqrt{\frac{45}{2}} = \frac{3\sqrt{10}}{2}.$$

$a = \frac{3\sqrt{10}}{2}$. **(3)**

www.mathswithmatt.co.uk

Question 4

Question 4

The first term of a geometric sequence is 6. The fourth term is $\frac{2}{9}$. The sequence is infinite. Find the sum of the associated series. [3]

Model solution

For a geometric sequence the fourth term is ar^3. This determines the common ratio, then the infinite-series formula applies because the ratio has modulus less than 1.

$$6r^3 = \frac{2}{9} \quad \Longrightarrow \quad r^3 = \frac{1}{27} \quad \Longrightarrow \quad r = \frac{1}{3}.$$

$$S_\infty = \frac{a}{1-r} = \frac{6}{1-\frac{1}{3}} = 9.$$

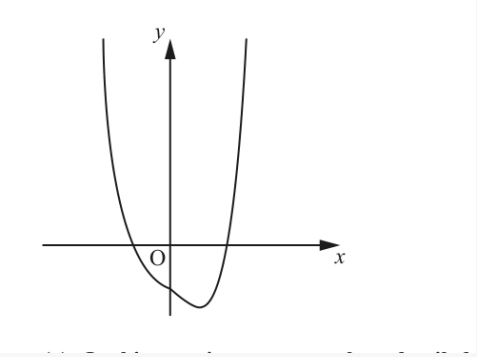
9. *(3)*

www.mathswithmatt.co.uk

Question 5

Question 5

The diagram shows the curve with equation $y = x^4 - 2^x$, for values of x close to zero.



(a) In this question you must show detailed reasoning. Show that the equation $x^4 - 2^x = 0$ has a root which lies between 1 and 2. [2]

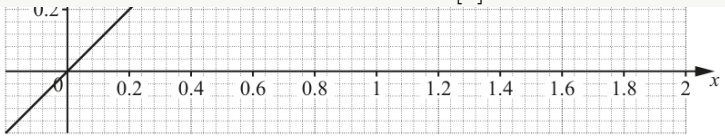
(b) Show that the equation $x^4 - 2^x = 0$ can be written in the form $x = 2^{0.25x}$ for positive values of x. [1]

(c) Use the iterative formula $x_{n+1} = 2^{0.25x_n}$, with $x_0 = 0.4$, to determine a root of $x^4 - 2^x = 0$ to 2 decimal places. [2]

(d) The diagram in the Printed Answer Booklet shows the line $y = x$ and the curve $y = 2^{0.25x}$. Sketch a cobweb or staircase diagram, on the diagram in the Printed Answer Booklet, for $x_{n+1} = 2^{0.25x_n}$, starting with $x_0 = 0.4$. Show at least two iterations. [2]

(e) Show that the equation $x^4 - 2^x = 0$ can be written in the form $x = 4\log_2 x$ for positive values of x. [1]

(f) In this question you must show detailed reasoning. Show that the iteration $x_{n+1} = 4\log_2 x_n$, with $x_0 = 1$, will not find a root of $x^4 - 2^x = 0$. [2]



5(e)

5(f)

Model solution

(a) Let $F(x) = x^4 - 2^x$. A continuous function which changes sign on an interval has a root in that interval.

$$F(1) = 1 - 2 = -1 < 0, \qquad F(2) = 16 - 4 = 12 > 0.$$

Since F changes sign between 1 and 2, it has a root between 1 and 2. **(2)**

(b) For positive x, rearrange before taking the fourth root.

$$x^4 = 2^x \implies x = (2^x)^{1/4} = 2^{0.25x}.$$

$x = 2^{0.25x}$. **(1)**

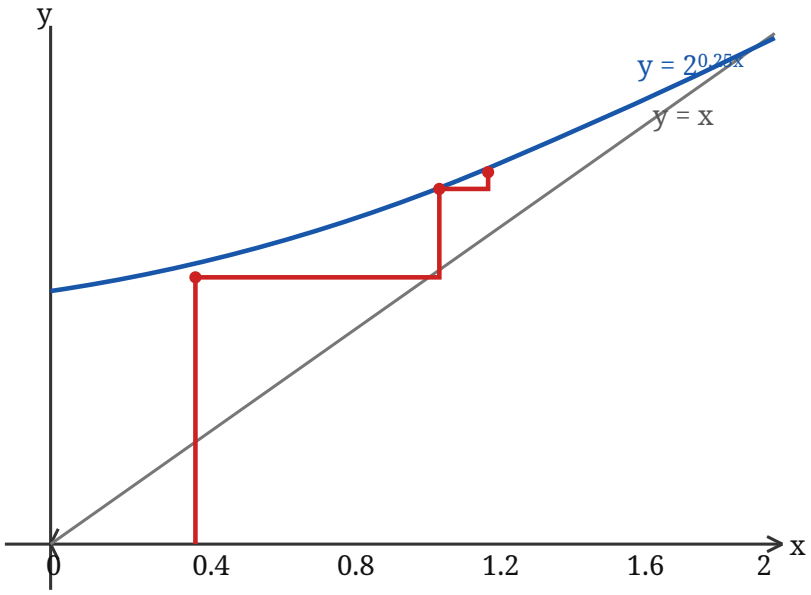
(c) Repeatedly substitute the latest value into the iteration. The values settle at the non-zero root.

n	0	1	2	3	4
x_n	0.4	1.0718	1.2042	1.2322	1.2382

$$x_5 = 2^{0.25(1.2382)} = 1.2395\ldots$$

The root is 1.24 (to 2 decimal places). **(2)**

(d) Start at $x = 0.4$ on the x-axis, move vertically to $y = 2^{0.25x}$, then horizontally to $y = x$; repeat. The first two iterates are 1.0718 and 1.2042, so the staircase heads towards 1.24.



A cobweb/staircase beginning $0.4 \to 1.0718 \to 1.2042$, with at least two vertical and two horizontal steps. **(2)**

(e) Taking logarithms base 2 is appropriate because the rearranged equation contains a power of 2.

$$x^4 = 2^x \implies \log_2(x^4) = \log_2(2^x) \implies 4\log_2 x = x.$$

$x = 4\log_2 x$. **(1)**

(f) Test the stated starting value. The next iterate is zero, but the following step would require a

logarithm of zero, which is undefined.

$$x_1 = 4\log_2(1) = 0, \qquad x_2 = 4\log_2(0) \text{ is undefined.}$$

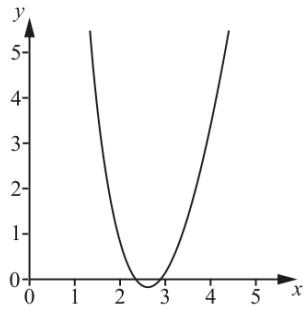
The iteration stops after $x_1 = 0$, so it cannot find a root. **(2)**

www.mathswithmatt.co.uk

Question 6

Question 6

The diagram shows the curve $y = f(x)$, where $f(x) = 13x - 34\ln x - 1.5$ for $x > 0$.
(a) A student attempts to locate a root of $f(x) = 0$ by using a spreadsheet to calculate values of $f(x)$ for positive integer values of x. Explain why the student's method may lead to the conclusion that $f(x) = 0$ has no roots. [1]
(b) Determine the exact value of the x-coordinate of the turning point of the curve. [2]



(a) A student attempts to locate a root of $f(x) = 0$ by using a spreadsheet to calculate the values of $f(x)$ for positive integer values of x.

Explain why the student's method may lead to the conclusion that $f(x) = 0$ has no roots. [

(b) Determine the exact value of the x-coordinate of the turning point of the curve. [

Model solution

(a) Checking only integers can miss crossings between consecutive integers. Here both roots lie between 2 and 3, while the function values at the checked integers are positive.

The two roots are between 2 and 3, so checking positive integers gives no sign change and may miss both roots. **(1)**

(b) A turning point occurs where the derivative is zero.

$$f'(x) = 13 - \frac{34}{x}.$$

$$13 - \frac{34}{x} = 0 \quad \Longrightarrow \quad x = \frac{34}{13}.$$

$x = \dfrac{34}{13}$. **(2)**

Question 7

Question 7

A group of scientists is studying how a population of insects grows in the laboratory. At the start of the study, there are 48 insects. At the end of the first month, there are 72 insects. At the end of the second month, there are 108 insects.
The group develops two models, P and Q, to model the population. The first model, P, is of the form $P_{t+1} = kP_t$, where k is a constant. P_0 denotes the number of insects at the start of the study. P_t denotes the number of insects at the end of month t, $t \geq 1$. Model P fits the data for $t = 0$, 1 and 2 exactly.
(a) Find k. [1]
(b) Use model P to predict the number of insects at the end of month 3. [1]
The second model, Q, is $Q = 48e^{0.4t}$, where t is the time, in months, after the start of the study and Q denotes the number of insects at time t.
(c) (i) By considering the given data when $t = 1$ and $t = 2$, assess whether model Q fits the data for these times. [2]
(ii) Show that model Q implies that the rate of growth of the population is proportional to the population at any time t. [2]
(iii) Show that, for integer values of t with $t \geq 1$, the population predicted by model Q will always be lower than the population predicted by model P. [3]

Model solution

(a) The recurrence multiplies the population by the same factor each month. Compare consecutive observed populations.

$$k = \frac{72}{48} = \frac{3}{2} = 1.5.$$

$k = 1.5$. **(1)**

(b) Apply this monthly multiplier once more.

$$P_3 = 1.5 \times 108 = 162.$$

162 *insects.* **(1)**

(c)(i) Substitute the two given times into the exponential model, then compare rather than expecting exact agreement.

$$Q(1) = 48e^{0.4} = 71.6\ldots \approx 72, \qquad Q(2) = 48e^{0.8} = 106.8\ldots \approx 107.$$

Model Q is close to, but does not exactly fit, the data 72 and 108. **(2)**

(c)(ii) Differentiate the exponential model. Expressing the derivative as a constant times Q proves proportionality.

$$\frac{dQ}{dt} = 48(0.4)e^{0.4t} = 0.4\left(48e^{0.4t}\right) = 0.4Q.$$

$\frac{dQ}{dt} = 0.4Q$, *so the growth rate is proportional to the population.* **(2)**

(c)(iii) Model P has monthly multiplier 1.5. Model Q has multiplier $e^{0.4}$ in one month. Both begin at 48, so the smaller multiplier remains smaller thereafter.

$$Q(t+1) = 48e^{0.4(t+1)} = Q(t)e^{0.4}, \qquad e^{0.4} = 1.4918\ldots < 1.5.$$

$$P_t = 48(1.5)^t, \qquad Q_t = 48\left(e^{0.4}\right)^t < 48(1.5)^t = P_t \quad (t \geq 1).$$

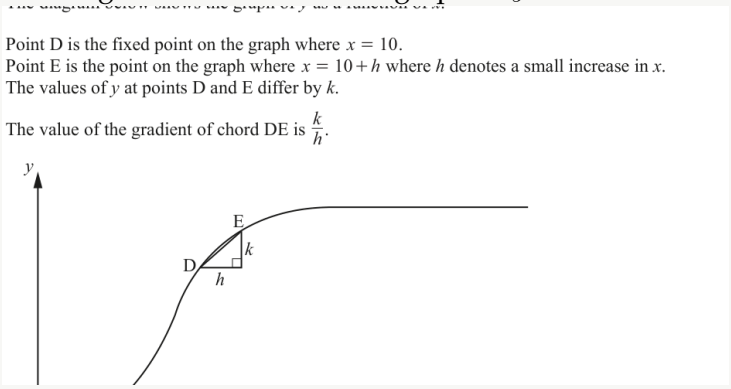
For every integer $t \geq 1$, $Q_t < P_t$. **(3)**

www.mathswithmatt.co.uk

Question 8

Question 8

The diagram below shows the graph of y as a function of x.



Point D is the fixed point on the graph where $x = 10$. Point E is the point on the graph where $x = 10 + h$, where h denotes a small increase in x. The values of y at D and E differ by k. The gradient of chord DE is $\frac{k}{h}$.

	A	B	C	D	E
1	h	$10 + h$	y	k	gradient
2	1	11	1992.8	197.5937	197.594
3	0.1	10.1	1817.54	22.33784	223.378
4	0.01	10.01	1797.46	2.259142	225.914
5	0.001	10.001	1795.43	0.226167	226.167
6	0.0001	10.0001	1795.22	0.022619	226.192
7	0.00001	10.00001	1795.2	0.002262	226.195

(a) Write down the value for cell B3. [1]
(b) Find the value for cell E3. Give your answer to 3 decimal places. [1]
(c) Deduce, to 1 decimal place, the value of the gradient of the graph at D. [1]

Model solution

(a) Column B contains $10 + h$.

$$10 + 0.1 = 10.1.$$

10.1. *(1)*

(b) The gradient is k/h, so use the row-three values.

$$\frac{22.33784}{0.1} = 223.3784.$$

223.378. *(1)*

(c) As h gets very small, the chord gradient approaches the tangent gradient. The final values approach $226.195\ldots$.

226.2. *(1)*

Question 9

Question 9

The function $f(x)$ is defined on all real numbers by $f(x) = x^3 + e^{3x}$.
(a) Differentiate $x^3 + e^{3x}$. [2]
(b) Write down the coordinates of the point where $y = f(x)$ crosses the y-axis. [1]
(c) Find the gradient of the curve of the inverse function, $y = f^{-1}(x)$, at the point where it crosses the x-axis. [2]

Model solution

(a) Use the power rule and the chain rule: differentiating e^{3x} introduces a factor of 3.

$$f'(x) = 3x^2 + 3e^{3x}.$$

$3x^2 + 3e^{3x}$. **(2)**

(b) On the y-axis, $x = 0$.

$$f(0) = 0^3 + e^0 = 1.$$

$(0, 1)$. **(1)**

(c) The inverse swaps coordinates, so its x-axis crossing corresponds to $(0, 1)$ on f. Inverse gradients are reciprocals.

$$f'(0) = 3(0)^2 + 3e^0 = 3, \qquad \left.\frac{dy}{dx}\right|_{f^{-1}} = \frac{1}{f'(0)} = \frac{1}{3}.$$

$\frac{1}{3}$. **(2)**

Question 10

Question 10

The magnitude of the vector $\begin{pmatrix} 4 \\ 1 \\ -x \end{pmatrix}$ is an integer. Determine all possible integer values of x. [6]

Model solution

The square of the magnitude must be a perfect square. Factor the difference of two squares to find all integer possibilities, rather than testing indefinitely.

$$\left| \begin{pmatrix} 4 \\ 1 \\ -x \end{pmatrix} \right|^2 = 4^2 + 1^2 + x^2 = 17 + x^2 = n^2,$$

where n is an integer. Hence

$$n^2 - x^2 = 17 \quad \Longrightarrow \quad (n - x)(n + x) = 17.$$

Since 17 is prime, the integer factor pairs are $(1, 17)$ and $(-1, -17)$:

$$n - x = 1,\ n + x = 17 \Rightarrow n = 9,\ x = 8;$$

$$n - x = -1,\ n + x = -17 \Rightarrow n = -9,\ x = -8.$$

$x = 8$ **or** $x = -8$. **(6)**

Question 11

Question 11

The nth term of a sequence is defined by $a_n = \dfrac{1}{\sqrt{n+1}-1} - \dfrac{1}{\sqrt{n+1}+1}$, for $n \geq 1$.

(a) Prove that every term in the sequence, a_n, is a rational number. [3]

(b) Determine whether the sequence is increasing, decreasing or neither. [2]

Model solution

(a) Combine the two fractions. The conjugate denominators are designed to remove the surd.

$$a_n = \frac{(\sqrt{n+1}+1)-(\sqrt{n+1}-1)}{(\sqrt{n+1}-1)(\sqrt{n+1}+1)} = \frac{2}{(n+1)-1} = \frac{2}{n}.$$

Because n is a positive integer, 2 and n are integers and $n \neq 0$. Thus $a_n = 2/n$ is rational.

Every term is rational. **(3)**

(b) Compare successive terms of the simplified expression.

$$a_{n+1} - a_n = \frac{2}{n+1} - \frac{2}{n} = -\frac{2}{n(n+1)} < 0 \qquad (n \geq 1).$$

The sequence is decreasing. **(2)**

www.mathswithm

Question 12

Question 12

(a) Show that for the point on the curve where $x = 0$, $\dfrac{3a - x}{a + x} = 3 \geq 0$, as given in line 10. [1]
(b) Use the equation of the curve given in line 8 to show that for all points on the curve where $x \neq 0$, $\dfrac{3a - x}{a + x} \geq 0$, as given in line 10. [1]
(c) Show that it follows that $3a - x \geq 0$ and $a + x > 0$, as given in lines 10 and 11, for values of $a > 0$. [3]
(d) Hence find, in terms of a where $a > 0$, the range of values of x for points on the curve $y^2 = \dfrac{x^2(3a - x)}{a + x}$. [1]
(e) Find the equation of the asymptote to the curve. [1]

Model solution

(a) Substitute $x = 0$ directly.

$$\frac{3a - 0}{a + 0} = \frac{3a}{a} = 3 \geq 0.$$

$\dfrac{3a - x}{a + x} = 3 \geq 0$ ***when*** $x = 0$. ***(1)***

(b) For $x \neq 0$, $x^2 > 0$, while every real y has $y^2 \geq 0$. Divide the curve equation by the positive factor x^2.

$$\frac{3a - x}{a + x} = \frac{y^2}{x^2} \geq 0.$$

$\dfrac{3a - x}{a + x} \geq 0$. ***(1)***

(c) The denominator cannot be zero. A non-negative quotient has numerator and denominator of the same sign (or a zero numerator). The both-negative case would require incompatible inequalities.

$$a + x \neq 0.$$

$$3a - x \leq 0 \text{ and } a + x < 0 \implies x \geq 3a \text{ and } x < -a.$$

This is impossible because $a > 0$, so $3a > -a$. Therefore the valid case is

$$3a - x \geq 0, \qquad a + x > 0.$$

$3a - x \geq 0$ ***and*** $a + x > 0$. ***(3)***

(d) Make x the subject of the two inequalities found above.

$$x \leq 3a, \qquad x > -a.$$

$-a < x \leq 3a$. ***(1)***

(e) The denominator is zero at $a + x = 0$, giving the vertical asymptote.

$x = -a$. ***(1)***

Question 13

Question 13

Use $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$, with suitable choices for A and B, to show that $\tan(3\theta) = \dfrac{t(3-t^2)}{1-3t^2}$, where $t = \tan\theta$, as given in line 25. [3]

Model solution

Write $3\theta = 2\theta + \theta$. First obtain $\tan 2\theta$ in terms of t, then apply the addition formula.

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2t}{1-t^2}.$$

$$\tan 3\theta = \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta} = \frac{\frac{2t}{1-t^2} + t}{1 - \frac{2t^2}{1-t^2}}.$$

$$\tan 3\theta = \frac{2t + t(1-t^2)}{1 - t^2 - 2t^2} = \frac{3t - t^3}{1-3t^2} = \frac{t(3-t^2)}{1-3t^2}.$$

$\tan(3\theta) = \dfrac{t(3-t^2)}{1-3t^2}$. ***(3)***

www.mathswithmatt.co.uk

Question 14

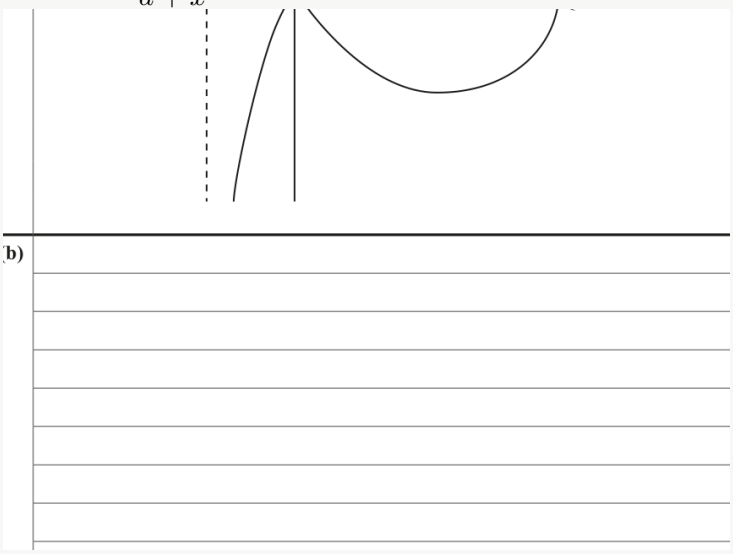
Question 14

(a) Draw and label a suitable triangle on the diagram in the Printed Answer Booklet to show that $\tan\theta = \frac{y}{x}$, as given in line 30. [1]

(b) Subtract $\frac{a(3-t^2)}{1+t^2}$ from $3a$ to show that $3a - x = \frac{4at^2}{1+t^2}$. [1]

(c) Find an expression, in terms of a and t, for $a + x$. [1]

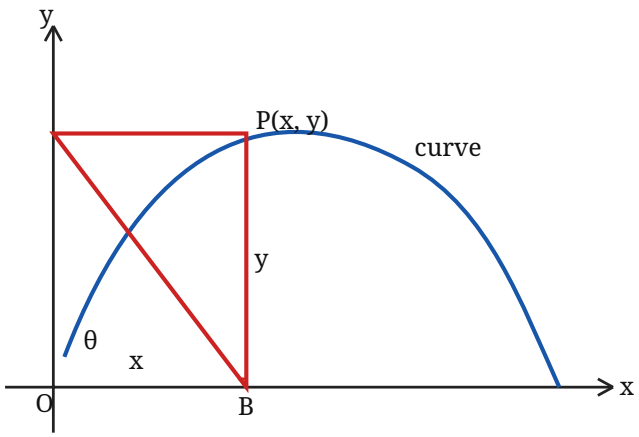
(d) Hence, or otherwise, show that the parametric equations given in lines 29 and 31 are equivalent to $y^2 = \frac{x^2(3a-x)}{a+x}$, as claimed in lines 8 and 33. [2]



Model solution

(a) Draw the right triangle from $P(x, y)$ vertically down to the x-axis. Its opposite side is y and adjacent side is x.

$$\tan\theta = \frac{\textit{opposite}}{\textit{adjacent}} = \frac{y}{x}.$$



A right triangle POB, with $PB = y$, $OB = x$, and the right angle at B. **(1)**

(b) Put the terms over the common denominator $1+t^2$.

$$3a - x = 3a - \frac{a(3-t^2)}{1+t^2} = \frac{3a(1+t^2) - a(3-t^2)}{1+t^2} = \frac{4at^2}{1+t^2}.$$

$3a - x = \dfrac{4at^2}{1+t^2}$. *(1)*

(c) Add a to the given expression for x over the same denominator.

$$a + x = a + \frac{a(3-t^2)}{1+t^2} = \frac{a(1+t^2) + a(3-t^2)}{1+t^2} = \frac{4a}{1+t^2}.$$

$a + x = \dfrac{4a}{1+t^2}$. *(1)*

(d) Divide the expressions from parts (b) and (c). Then use $t = y/x$ from the right triangle.

$$\frac{3a-x}{a+x} = \frac{\frac{4at^2}{1+t^2}}{\frac{4a}{1+t^2}} = t^2 = \frac{y^2}{x^2}.$$

$$y^2 = \frac{x^2(3a-x)}{a+x}.$$

The parametric equations are equivalent to $y^2 = \dfrac{x^2(3a-x)}{a+x}$. *(2)*

www.mathswithm