Maths with Matt

Physics and maths resources

OCR AS Level Further Mathematics A

H235/Y533 - Mechanics

Wednesday 15 June 2022

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Wednesday 15 June 2022 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Contents

Question 1 2

Question 2 3

Question 3 4

Question 4 5

Question 5 6

Question 6 7

Question 7 8

Question 8 10

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Question 1

Question 1

Two stones, A and B, are sliding along the same straight line on a horizontal sheet of ice. Stone A, of mass 50 kg, is moving with a constant velocity of 2.1 m s^{-1} towards stone B. Stone B, of mass 70 kg, is moving with a constant velocity of 0.8 m s^{-1} towards stone A.
A and B collide directly. Immediately after their collision stone A's velocity is 0.35 m s^{-1} in the same direction as its velocity before the collision.
(a) Find the speed of stone B immediately after the collision. [2]
(b) Find the coefficient of restitution for the collision. [2]
(c) Find the total loss of kinetic energy caused by the collision. [3]
(d) Explain whether the collision was perfectly elastic. [1]

Model solution

Take A's initial direction as positive. The ice is horizontal, so there is no external horizontal impulse and momentum is conserved.

(a) Before the collision B has velocity -0.8 m s^{-1}. Let its final velocity be v_B.

$$50(2.1) + 70(-0.8) = 50(0.35) + 70v_B,$$

$$49 = 17.5 + 70v_B, \qquad v_B = 0.45.$$

The speed of B is 0.450 m s^{-1}. **(2)**

(b) The coefficient of restitution is the speed of separation divided by the speed of approach. With the chosen direction, A and B separate with velocities 0.35 and 0.45 respectively.

$$e = \frac{0.45 - 0.35}{2.1 - (-0.8)} = \frac{0.10}{2.9} = \frac{1}{29}.$$

$e = \dfrac{1}{29} \approx 0.0345.$ **(2)**

(c) Calculate the total kinetic energy before and after the collision, then subtract.

$$E_{\text{i}} = \tfrac{1}{2}(50)(2.1)^2 + \tfrac{1}{2}(70)(0.8)^2 = 132.65 \text{ J},$$

$$E_{\text{f}} = \tfrac{1}{2}(50)(0.35)^2 + \tfrac{1}{2}(70)(0.45)^2 = 10.15 \text{ J}.$$

$$\text{loss} = 132.65 - 10.15 = 122.5 \text{ J}.$$

The total loss of kinetic energy is 123 J (or 122.5 J). **(3)**

(d) A perfectly elastic collision has no kinetic-energy loss (equivalently, $e = 1$).

No. Kinetic energy is lost, so the collision is not perfectly elastic. **(1)**

Question 2

Question 2

A hockey puck of mass 0.2 kg is sliding down a rough slope which is inclined at $10°$ to the horizontal. At the instant that its velocity is 14 m s^{-1} directly down the slope it is hit by a hockey stick. Immediately after it is hit its velocity is 24 m s^{-1} directly up the slope.

(a) Find the magnitude of the impulse exerted by the hockey stick on the puck. [2]

After it has been hit, the puck first comes to instantaneous rest when it has travelled 15 m up the slope. While the puck is moving up the slope, the resistance to its motion has constant magnitude R N.

(b) Use an energy method to determine the value of R. [5]

Model solution

Take up the slope as positive. Impulse equals change in momentum, and the initial velocity is negative in this convention.

(a)

$$I = m(v-u) = 0.2\big(24-(-14)\big) = 7.6 \text{ N s}.$$

The impulse has magnitude 7.6 *N s.* *(2)*

(b) Energy is convenient because the puck's final speed is zero and both the vertical rise and work against the constant resistance can be found. Its rise is $15\sin 10°$ *m.*

$$\textit{initial KE} = \tfrac{1}{2}(0.2)(24)^2 = 57.6 \text{ J},$$

$$\textit{gain in PE} = 0.2(9.8)(15\sin 10°), \qquad \textit{work against resistance} = 15R.$$

$$57.6 = 0.2(9.8)(15\sin 10°) + 15R.$$

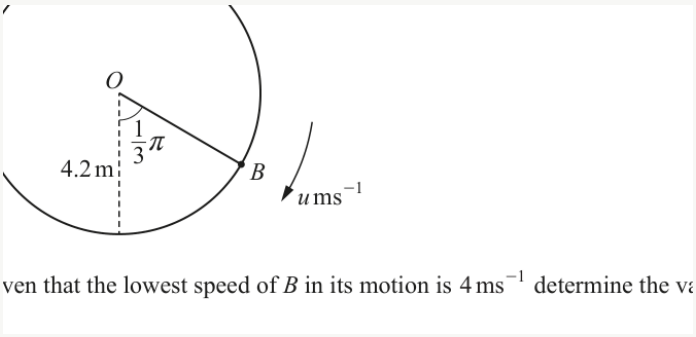
$$R = 3.4996\ldots$$

$R = 3.50$ *N.* *(5)*

Question 3

Question 3

A smooth wire is shaped into a circle of radius 4.2 m which is fixed in a vertical plane with its centre at a point O. A small bead B is threaded onto the wire. B is held so that OB makes an angle of $\frac{1}{3}\pi$ radians with the downwards vertical through O. B is projected downwards along the wire with initial speed u m s^{-1} (see diagram). In its subsequent motion B describes complete circles about O.



ven that the lowest speed of B in its motion is $4\,\text{ms}^{-1}$ determine the va

Given that the lowest speed of B in its motion is 4 m s^{-1}, determine the value of u. [5]

Model solution

The wire is smooth, so mechanical energy is conserved. The bead is slowest at the top of the circle, where its gravitational potential energy is greatest. Take the lowest point as the zero level of potential energy.

$$h_{\text{start}} = 4.2\left(1-\cos\frac{\pi}{3}\right) = 2.1 \text{ m}, \qquad h_{\text{top}} = 2(4.2) = 8.4 \text{ m}.$$

At the top the speed is 4 m s^{-1}, so equate the energy at the initial position to the energy at the top.

$$m(9.8)(2.1) + \tfrac{1}{2}mu^2 = m(9.8)(8.4) + \tfrac{1}{2}m(4)^2.$$

$$20.58m + \tfrac{1}{2}mu^2 = 90.32m, \qquad u^2 = 139.48.$$

$u = 11.8$ **m s^{-1}.** **(5)**

Question 4

Question 4

A cyclist is riding a bicycle along a straight road which is inclined at an angle of $4°$ to the horizontal. The cyclist is working at a constant rate of 250 W. The combined mass of the cyclist and bicycle is 80 kg and the resistance to their motion is a constant 70 N.
Determine the maximum constant speed at which the cyclist can ride the bicycle

- up the hill, and
- down the hill.

[5]

Model solution

At constant speed the resultant force along the slope is zero. Since power $P = Fv$, the cyclist's driving force at speed v is $F = 250/v$. The limiting speed in each direction occurs when this driving force balances the opposing forces.
For motion up the hill, both resistance and the component of weight down the slope oppose the motion.

$$\frac{250}{v} - 70 - 80(9.8)\sin 4° = 0,$$

$$v = \frac{250}{70 + 80(9.8)\sin 4°} = 2.0049\ldots$$

Maximum speed up the hill: 2.00 *m s*$^{-1}$. *(3)*

For motion down the hill, the component of weight assists the cyclist but resistance opposes the motion.

$$\frac{250}{v} + 80(9.8)\sin 4° - 70 = 0,$$

$$v = \frac{250}{70 - 80(9.8)\sin 4°} = 16.329\ldots$$

Maximum speed down the hill: 16.3 *m s*$^{-1}$. *(2)*

Question 5

Question 5

One end of a light inextensible string of length 3.5 m is attached to a fixed point O on a smooth horizontal plane. The other end of the string is attached to a particle P of mass 0.45 kg. P moves with constant speed in a circular path on the plane with the string taut.
The string will break if the tension in it exceeds 70 N.
Determine the minimum possible time in which P can describe a complete circle about O. [4]

Model solution

The shortest period needs the greatest permitted speed. On the horizontal plane the tension provides the centripetal force, so use the breaking tension as the limiting tension.

$$70 = \frac{0.45v^2}{3.5}, \qquad v^2 = \frac{70(3.5)}{0.45}.$$

A complete circle has circumference $2\pi r$, hence $T = 2\pi r/v$.

$$T_{\min} = \frac{2\pi(3.5)}{\sqrt{70(3.5)/0.45}} = 0.94247\ldots \text{ s}.$$

The minimum possible time is 0.942 s. **(4)**

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Question 6

Question 6

A particle moves in a straight line with constant acceleration a. Its initial velocity is u and at time t its velocity is v.
It is assumed that v depends only on u, a and t.
(a) Assuming that this dependency is of the form $u^\alpha a^\beta t^\gamma$, use dimensional analysis to find α and γ in terms of β. [5]
(b) By noting that the graph of v against t must be a straight line, determine the possible values of β. [2]
You may assume that the units of the given quantities are the corresponding SI units.
(c) By considering v when $t = 0$ seconds and when $t = 1$ second, derive the equation of motion $v = u + at$, explaining your reasoning. [3]

Model solution

(a) Dimensions must agree on both sides of a physical relation. Velocity has dimensions LT^{-1}, while $[u] = LT^{-1}$, $[a] = LT^{-2}$ and $[t] = T$.

$$[u^\alpha a^\beta t^\gamma] = (LT^{-1})^\alpha (LT^{-2})^\beta T^\gamma = L^{\alpha+\beta} T^{-\alpha-2\beta+\gamma}.$$

Equating the powers of L and T with $[v] = LT^{-1}$ gives

$$\alpha + \beta = 1, \qquad -\alpha - 2\beta + \gamma = -1.$$

$$\alpha = 1 - \beta, \qquad \gamma = \beta.$$

$\alpha = 1 - \beta$ *and* $\gamma = \beta$. *(5)*

(b) Thus a possible term in v is $u^{1-\beta} a^\beta t^\beta$. A straight-line graph can contain only a constant term or a term proportional to t, so t^β must be t^0 or t^1.

$$\beta = 0 \quad \text{or} \quad \beta = 1.$$

$\beta = 0$ *or* $\beta = 1$. *(2)*

(c) The two allowed types of term imply $v = k_1 u + k_2 at$, for constants k_1, k_2. At $t = 0$, the velocity is its initial value u, so $k_1 = 1$. At $t = 1$, constant acceleration means the velocity has increased by a, giving $v = u + a$, so $k_2 = 1$.

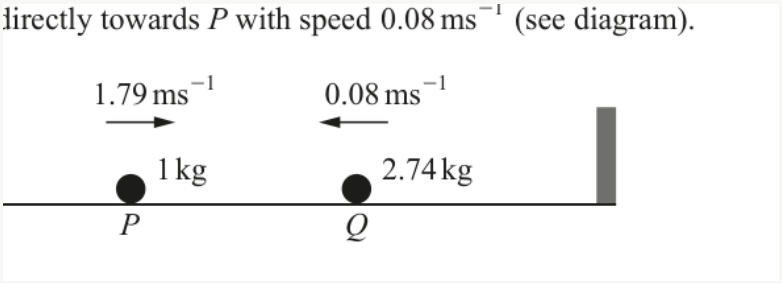
$$v = u + at.$$

$v = u + at$. *(3)*

Question 7

Question 7

Two particles, P and Q, are on a smooth horizontal floor. P, of mass 1 kg, is moving with speed 1.79 m s^{-1} directly towards a vertical wall. Q, of mass 2.74 kg, is between P and the wall, moving directly towards P with speed 0.08 m s^{-1} (see diagram).



P and Q collide directly and the coefficient of restitution for this collision is denoted by e.
(a) Show that after this collision the speed of Q is given by $0.42 + 0.5e$ m s^{-1}. [5]
After this collision, Q then goes on to collide directly with the wall. The coefficient of restitution for the collision between Q and the wall is also e. There is then no subsequent collision between P and Q.
(b) Determine the range of possible values of e. [7]

Model solution

Take the direction towards the wall as positive. Let the velocities after the P--Q collision be v_P and v_Q. The floor is smooth, so horizontal momentum is conserved.
(a)

$$1(1.79) + 2.74(-0.08) = v_P + 2.74v_Q,$$

$$1.5708 = v_P + 2.74v_Q. \qquad (1)$$

The restitution equation uses relative speed of separation over relative speed of approach:

$$v_Q - v_P = e\big(1.79 - (-0.08)\big) = 1.87e, \qquad v_P = v_Q - 1.87e. \qquad (2)$$

Substitute (2) into (1).

$$1.5708 = v_Q - 1.87e + 2.74v_Q = 3.74v_Q - 1.87e,$$

$$v_Q = \frac{1.5708 + 1.87e}{3.74} = 0.42 + 0.5e.$$

The speed of Q is $0.42 + 0.5e$ m s^{-1}. *(5)*

(b) First obtain P's velocity after the first collision.

$$v_P = v_Q - 1.87e = 0.42 - 1.37e.$$

When Q strikes the wall it rebounds, so its velocity afterwards is

$$w_Q = -e(0.42 + 0.5e).$$

There is no second P--Q collision. Since P is behind Q, this requires P not to catch Q after Q rebounds; therefore $v_P \le w_Q$.

$$0.42 - 1.37e \le -e(0.42 + 0.5e),$$

$$e^2 - 1.9e + 0.84 \le 0.$$

The critical values are found from the corresponding quadratic equation:

$$e = \frac{1.9 \pm \sqrt{1.9^2 - 4(0.84)}}{2} = 0.7,\ 1.2.$$

Thus the inequality gives $0.7 \leq e \leq 1.2$. *A coefficient of restitution must also satisfy* $0 \leq e \leq 1$.

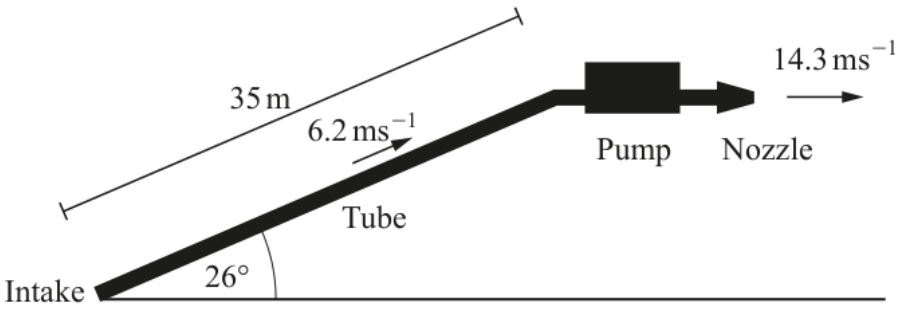
$0.7 \leq e \leq 1$. **(7)**

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Question 8

Question 8

As part of an industrial process a single pump causes the intake of a liquid chemical to the bottom end of a tube, draws it up the tube and then discharges it through a nozzle at the top end of the tube.
The tube is straight and narrow, 35 m long and inclined at an angle of $26°$ to the horizontal. The chemical arrives at the intake at the bottom end of the tube with a speed of 6.2 m s^{-1}. At the top end of the tube the chemical is discharged horizontally with a speed of 14.3 m s^{-1} (see diagram). In total, the pump discharges 1500 kg of chemical through the nozzle each hour.



In order to model the changes to the mechanical energy of the chemica

In order to model the changes to the mechanical energy of the chemical during the entire process of intake, drawing and discharge, the following modelling assumptions are made.

- At any instant the total resistance to the motion of all the liquid in the tube is 40 N.
- All other resistances to motion are ignored.
- The liquid in the tube moves at a constant speed of 6.2 m s^{-1}.

(a) State one other modelling assumption which is required to model the changes to the mechanical energy of the liquid with the given information. [1]
(b) Determine the power at which the pump is working, according to the model. [5]
When the power at which the pump is working is measured it is in fact found to be 450 W.
(c) (i) Find the difference between the total amount of energy output by the pump each hour and the total amount of mechanical energy gained by the chemical each hour. [2]
(ii) Give one reason why the model underestimates the power of the engine. [1]

Model solution

(a) To use the given intake speed as the starting kinetic energy, its direction must be known.

Assume that the incoming chemical has velocity directed into the tube. ***(1)***

(b) Work over one hour. The pump must supply the chemical's increases in kinetic and gravitational potential energy, and must do work against the stated resistance. The vertical rise is $35 \sin 26°$ *m.*

$$\Delta KE = \tfrac{1}{2}(1500)\left(14.3^2 - 6.2^2\right) = 124537.5 \text{ J},$$

$$\Delta PE = (1500)(9.8)(35 \sin 26°) = 225541.955\ldots \text{ J}.$$

In one hour, liquid travels through the tube at 6.2 *m* s^{-1}*, so the work done against resistance is*

$$W_R = 40(6.2)(3600) = 892800 \text{ J}.$$

$$P = \frac{\Delta KE + \Delta PE + W_R}{3600} = \frac{124537.5 + 225541.955\ldots + 892800}{3600} = 344.97\ldots \text{ W}.$$

The pump power is 345 *W.* *(5)*

(c)(i) The requested mechanical-energy gain excludes the energy dissipated against resistance. The measured output in one hour is 450×3600 *J.*

$$\textit{difference} = 450(3600) - \left(124537.5 + 225541.955\ldots\right) = 1269920\ldots \text{ J}.$$

The difference is 1.27×10^6 *J, or* 1270 *kJ.* *(2)*

(c)(ii) The actual process can require energy for effects excluded by the model.

For example, there may be additional resistance to the liquid's motion (such as resistance at the nozzle), so extra work is required. *(1)*

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