Maths with Matt

Physics and maths resources

OCR AS Level Further Mathematics A

H235/Y533 - Mechanics

Wednesday 14 June 2023

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Wednesday 14 June 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Two particles A, of mass m kg, and B, of mass $3m$ kg, are connected by a light inextensible string and placed together at rest on a smooth horizontal surface with the string slack. A is projected along the surface, directly away from B, with a speed of 2.4 m s^{-1}.
(a) Find the speed of B immediately after the string becomes taut. [2]
(b) Find, in terms of m, the magnitude of the impulse exerted on B as a result of the string becoming taut. [2]

Model solution

(a) When the string becomes taut, the particles have a common velocity along the line of the string. There is no external horizontal impulse, so horizontal momentum is conserved.

$$m(2.4) + 3m(0) = (m + 3m)v.$$

$$2.4m = 4mv, \qquad v = 0.6.$$

The speed of B is 0.6 m s^{-1}. **(2)**

(b) Impulse is the change in momentum. Particle B starts at rest and then moves at 0.6 m s^{-1}.

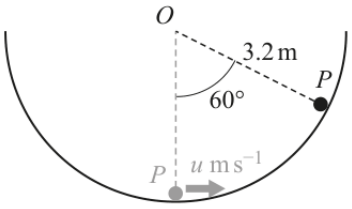
$$I_B = 3m(0.6) - 3m(0) = 1.8m.$$

The magnitude of the impulse on B is $1.8m$ N s. **(2)**

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Question 2

Question 2



A small body P of mass 3 kg is at rest at the lowest point of th
shell of radius 3.2 m and centre O.

P is projected horizontally with a speed of u m s^{-1}. When P fir
makes an angle of 60° with the downward vertical through O.

A small body P of mass 3 kg is at rest at the lowest point of the inside of a smooth hemispherical shell of radius 3.2 m and centre O. P is projected horizontally with a speed of u m s^{-1}. When P first comes to instantaneous rest OP makes an angle of 60° with the downward vertical through O.
(a) Find the value of u. [4]
(b) State one assumption made in modelling the motion of P. [1]

Model solution

(a) The shell is smooth, so use conservation of mechanical energy. The body rises through the vertical height $3.2 - 3.2\cos 60°$. *At the turning point its kinetic energy is zero.*

$$\frac{1}{2}(3)u^2 = 3g\left(3.2 - 3.2\cos 60°\right) = 3(9.8)(1.6) = 47.04.$$

$$1.5u^2 = 47.04, \qquad u^2 = 31.36, \qquad u = 5.6.$$

$u = 5.6$ ***m s***$^{-1}$. ***(4)***

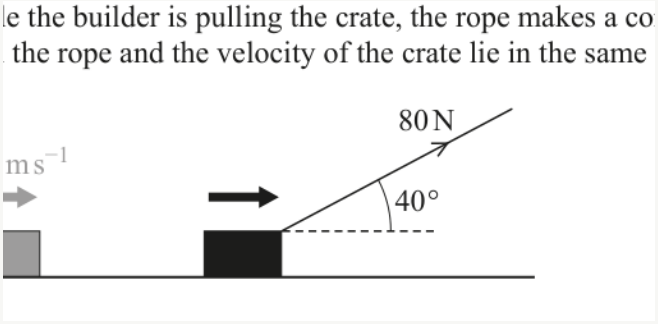
(b) One modelling assumption, in addition to the stated smoothness, is that air resistance is negligible.

Air resistance is negligible. *(1)*

Question 3

Question 3

A crate of mass 45 kg is sliding with a speed of 0.8 m s^{-1} in a straight line across a smooth horizontal floor. One end of a light inextensible rope is attached to the crate. At a certain instant a builder takes the other end of the rope and starts to pull, applying a constant force of 80 N for 5 seconds. While the builder is pulling the crate, the rope makes a constant angle of $40°$ above the horizontal. Both the rope and the velocity of the crate lie in the same vertical plane (see diagram).



It may be assumed that there is no resistance to the motion of the crate.
(a) Determine the work done by the builder in pulling the crate. [4]
(b) (i) Find the kinetic energy of the crate at the instant when the builder stops pulling the crate. [2]
(ii) Explain why the answers to part (a) and part (b)(i) are not equal. [1]
(c) Find the average power developed by the builder in pulling the crate. [1]
(d) Calculate the total impulse exerted on the crate by the builder. [2]

Model solution

(a) Only the horizontal component of the pull accelerates the crate, since it stays on the horizontal floor. First find the acceleration and hence the distance travelled in the five seconds.

$$80\cos 40° = 45a, \qquad a = 1.36185\ldots \text{ m s}^{-2}.$$

$$s = ut + \frac{1}{2}at^2 = 0.8(5) + \frac{1}{2}(1.36185\ldots)(5)^2 = 21.0232\ldots \text{ m}.$$

$$W = (80\cos 40°)s = 1288.38\ldots \text{ J}.$$

The work done is 1.29×10^3 *J.* *(4)*

(b)(i) Use constant-acceleration motion to find the final speed, then substitute it into $\frac{1}{2}mv^2$.

$$v = 0.8 + (1.36185\ldots)(5) = 7.60928\ldots \text{ m s}^{-1},$$

$$E_k = \frac{1}{2}(45)(7.60928\ldots)^2 = 1302.78\ldots \text{ J}.$$

The kinetic energy is 1.30×10^3 *J.* *(2)*

(b)(ii) Work done equals the increase in kinetic energy, not the final kinetic energy itself. The crate already had kinetic energy when the pull began.

The crate has initial kinetic energy, so the work done is its increase in kinetic energy. *(1)*

(c) Average power is total work done divided by the time taken.

$$\overline{P} = \frac{1288.38\ldots}{5} = 257.675\ldots \text{ W}.$$

The average power is 258 *W.* *(1)*

(d) The impulse is force multiplied by time; again, only the horizontal component changes the crate's momentum.

$$I = (80\cos 40^\circ)(5) = 306.418\ldots \text{ N s}.$$

The total impulse is 306 *N s.* *(2)*

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Question 4

Question 4

A rower is rowing a boat in a straight line across a lake. The combined mass of the rower, boat and oars is 240 kg. The maximum power that the rower can generate is 450 W.
In a model of the motion of the boat it is assumed that the total resistance to the motion of the boat is 150 N at any instant when the boat is in motion.
(a) Find the maximum possible acceleration of the boat, according to the model, at an instant when its speed is 0.5 m s^{-1}. [2]
At one stage in its motion the boat is travelling at a constant speed and the rower is generating power at an average rate of 210 W, which is assumed to be constant. The boat passes a pole and then, after travelling 350 m, a second pole.
(b) Determine how long it takes, according to the model, for the boat to travel between the two poles. [4]
(c) State a reason why the assumption that the rower's generated power is constant may be unrealistic. [1]

Model solution

(a) The clue is that power and speed are given, so use $P = Fv$ to find the maximum driving force. Resolve along the direction of motion using Newton's second law.

$$F = \frac{450}{0.5} = 900 \text{ N}, \qquad 900 - 150 = 240a.$$

$$a = 3.125 \text{ m s}^{-2}.$$

The maximum acceleration is 3.13 m s^{-2}. **(2)**

(b) At constant speed the resultant force is zero, so the driving force equals the 150 N resistance. Use this in $P = Fv$, then use $t = d/v$.

$$210 = 150v, \qquad v = 1.4 \text{ m s}^{-1}.$$

$$t = \frac{350}{1.4} = 250 \text{ s}.$$

The time is 250 s (4 minutes 10 seconds). **(4)**

(c) A human rower cannot generally deliver precisely the same power continuously; for example, fatigue can reduce the power output.

The rower may tire, so the generated power may decrease. **(1)**

Question 5

Question 5

Two identical spheres, A and B, each of mass 4 kg, are moving directly towards each other along the same straight line on a smooth horizontal surface until they collide. Before they collide, the speeds of A and B are 5 m s^{-1} and 3 m s^{-1} respectively. Immediately after they collide, the speed of A is 2 m s^{-1} and its direction of motion has been reversed.

(a) (i) Determine the velocity of B immediately after A and B collide. [3]

(ii) Show that the coefficient of restitution between A and B is $\frac{3}{4}$. [2]

(iii) Calculate the total loss of kinetic energy due to this collision. [2]

Sphere B goes on to strike a fixed wall directly. As a result of this collision B moves along the same straight line with a speed of 4 m s^{-1}.

(b) Find the coefficient of restitution between B and the wall, stating whether the collision between B and the wall is perfectly elastic. [2]

(c) Determine the magnitude of the impulse that B exerts on A the next time that they collide. [5]

Model solution

Take the initial direction of A as positive. Thus initially $u_A = 5$, $u_B = -3$, and after the first collision $v_A = -2$, all in m s^{-1}.

(a)(i) Conservation of momentum applies because there is no external horizontal impulse during the collision.

$$4(5) + 4(-3) = 4(-2) + 4v_B.$$

$$8 = -8 + 4v_B, \qquad v_B = 4.$$

B has velocity 4 m s^{-1}, in A's original direction. **(3)**

(a)(ii) The coefficient of restitution is the speed of separation divided by the speed of approach. Keeping the signed velocities makes the directions explicit.

$$e = \frac{v_B - v_A}{u_A - u_B} = \frac{4 - (-2)}{5 - (-3)} = \frac{6}{8} = \frac{3}{4}.$$

$e = \frac{3}{4}$, as required. **(2)**

(a)(iii) Calculate total kinetic energy before and after the collision, then subtract.

$$E_{\text{before}} = \frac{1}{2}(4)(5^2) + \frac{1}{2}(4)(3^2) = 68 \text{ J},$$

$$E_{\text{after}} = \frac{1}{2}(4)(2^2) + \frac{1}{2}(4)(4^2) = 40 \text{ J}.$$

$$\text{loss} = 68 - 40 = 28 \text{ J}.$$

The total loss of kinetic energy is 28 J. **(2)**

(b) Just before the wall, B's speed is 4 m s^{-1}, and its rebound speed is also 4 m s^{-1}. For a fixed wall, e is rebound speed divided by approach speed.

$$e = \frac{4}{4} = 1.$$

$e = 1$; the collision is perfectly elastic. **(2)**

(c) After rebounding from the wall, B has velocity -4 m s^{-1}, while A still has velocity -2 m s^{-1}.

Use momentum conservation and the same $e = \frac{3}{4}$ for their next collision. Let their new velocities be V_A and V_B.

$$4(-2) + 4(-4) = 4V_A + 4V_B, \qquad V_A + V_B = -6,$$

$$\frac{3}{4} = \frac{V_B - V_A}{(-2) - (-4)}, \qquad V_B - V_A = \frac{3}{2}.$$

$$V_B = -2.25, \qquad V_A = -3.75.$$

The impulse exerted by B on A is the change in A's momentum.

$$I_A = 4\big((-3.75) - (-2)\big) = -7 \text{ N s}.$$

The magnitude of the impulse is 7 N s. **(5)**

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Question 6

Question 6

The physical quantity pressure, denoted by P, can be calculated using the formula $P = \frac{F}{A}$ where F is a force and A is an area.
(a) Find the dimensions of P. [1]
An object of mass m is moving on a smooth horizontal surface subject to a system of forces which begin to act at time $t = 0$. The initial velocity of the object is u and its velocity and acceleration at time t are denoted by v and a respectively. The object exerts a pressure P on the surface. The total work done by the forces is denoted by W.
A Mathematics class suggests three formulae to model the quantity W.
The first suggested formula is $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 + mP$.
(b) Use dimensional analysis to show that this formula cannot be correct. [2]
The second suggested formula is $W = ku^a v^b t^c$ where k is a dimensionless constant.
(c) Use dimensional analysis to show that this formula cannot be correct for any values of a, b and c. [2]
The third suggested formula is $W = ku^a a^b m^c t^d$ where k is a dimensionless constant.
(d) (i) Explain why it is not possible to use dimensional analysis to determine the values of a, b, c and d for the third suggested formula. [1]
(ii) Given that $a = 3$, use dimensional analysis to determine the values of b, c and d for the third suggested formula. [3]
(iii) By considering what the formula predicts for large values of t, explain why the formula derived in part (d)(ii) is likely to be incorrect. [1]

Model solution

(a) Pressure is force per unit area. Since $[F] = MLT^{-2}$ and $[A] = L^2$, divide their dimensions.

$[P] = ML^{-1}T^{-2}$. *(1)*

(b) Work and each kinetic-energy term have dimensions ML^2T^{-2}. However, the final term has different dimensions.

$$[mP] = M\left(ML^{-1}T^{-2}\right) = M^2L^{-1}T^{-2}.$$

Since $[mP] \neq [W]$, the formula is dimensionally inconsistent and cannot be correct. *(2)*

(c) Write the dimensions of the proposed right-hand side. It contains only speeds and time, so it has no mass dimension.

$$[ku^a v^b t^c] = (LT^{-1})^a (LT^{-1})^b T^c = L^{a+b}T^{-a-b+c}.$$

The right-hand side has no M factor, whereas $[W] = ML^2T^{-2}$; therefore no values of a, b, c can make the formula correct. *(2)*

(d)(i) Dimensional analysis gives one equation for each of the three base dimensions M, L, and T, but there are four unknown indices.

There are only three independent dimensional equations for four unknowns, so the indices cannot all be determined. *(1)*

(d)(ii) Here the stated $a = 3$ means the exponent of u is known. Equate dimensions on both sides.

$$ML^2T^{-2} = (LT^{-1})^3 (LT^{-2})^b M^c T^d = M^c L^{3+b} T^{-3-2b+d}.$$

$$c = 1, \qquad 3 + b = 2, \qquad -3 - 2b + d = -2.$$

$$b = -1, \qquad c = 1, \qquad d = -1.$$

$b = -1,\ c = 1,\ d = -1.$ **(3)**

(d)(iii) Substitution gives $W = \frac{ku^3m}{at}$. For large t, this predicts that the total work becomes very small, whereas total work done should not decrease merely because more time has passed.

The formula predicts $W \to 0$ as t becomes large, so it is unlikely to be correct. **(1)**

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Question 7

Question 7

Two identical light, inextensible strings S_1 and S_2 are each of length 5 m. Two identical particles P and Q are each of mass 1.5 kg.
One end of S_1 is attached to P. The other end of S_1 is attached to a fixed point A on a smooth horizontal plane. P moves with constant speed in a horizontal circular path with A as its centre (see Fig. 1).
One end of S_2 is attached to Q. The other end of S_2 is attached to a fixed point B. Q moves with constant speed in a horizontal circular path around a point O which is vertically below B. At any instant, BQ makes an angle of θ with the downward vertical through B (see Fig. 2).

Fig. 1

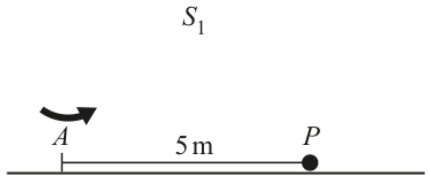
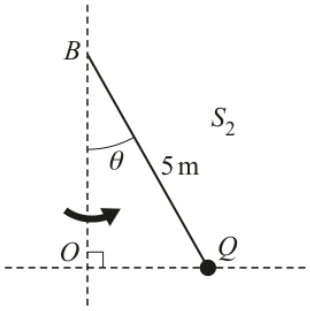


Fig. 2



(a) Given that the angular speed of P is the same as the angular speed of Q, show tha tensions in S_1 and S_2 have the same magnitude.

(a) Given that the angular speed of P is the same as the angular speed of Q, show that the tensions in S_1 and S_2 have the same magnitude. [3]
(b) You are given instead that the kinetic energy of P is 39.2 J and that this is the same as the kinetic energy of Q. Determine the difference between the times taken by P and Q to complete one revolution. Give your answer in an exact form. [7]

Model solution

(a) For both particles, use Newton's second law towards the centre of the horizontal circular path. Let their common angular speed be ω.

$$T_{S_1} = 1.5(5)\omega^2 = 7.5\omega^2.$$

For Q, the radius of its circular path is $5\sin\theta$. The horizontal component of its tension supplies the centripetal force.

$$T_{S_2}\sin\theta = 1.5(5\sin\theta)\omega^2,$$

$$T_{S_2} = 7.5\omega^2 = T_{S_1}.$$

The tensions in S_1 and S_2 have the same magnitude. **(3)**

(b) Equal masses and equal kinetic energies mean that P and Q have the same speed v. First find this speed from the given kinetic energy.

$$\frac{1}{2}(1.5)v^2 = 39.2, \qquad v^2 = \frac{784}{15}, \qquad v = \frac{28\sqrt{15}}{15}.$$

For P, the radius is 5 m, so its period is circumference divided by speed.

$$T_P = \frac{2\pi(5)}{v} = \frac{5\sqrt{15}\pi}{14}.$$

To find Q's radius, resolve its tension vertically and horizontally. The vertical component balances its weight, and the horizontal component supplies the centripetal force.

$$T_{S_2}\cos\theta = 1.5g,$$

$$T_{S_2}\sin\theta = 1.5\frac{v^2}{5\sin\theta}.$$

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{5g} = \frac{784/15}{5(9.8)} = \frac{16}{15}.$$

Using $\sin^2\theta = 1 - \cos^2\theta$ gives an equation in $\cos\theta$.

$$15(1-\cos^2\theta) = 16\cos\theta,$$

$$15\cos^2\theta + 16\cos\theta - 15 = 0,$$

$$(5\cos\theta - 3)(3\cos\theta + 5) = 0.$$

$$\cos\theta = \frac{3}{5}, \qquad \sin\theta = \frac{4}{5}.$$

Therefore Q's circular radius is $5\sin\theta = 4$ m.

$$T_Q = \frac{2\pi(4)}{v} = \frac{4\sqrt{15}\pi}{14}.$$

$$T_P - T_Q = \frac{\sqrt{15}\pi}{14}.$$

The difference between the times is $\frac{\sqrt{15}\pi}{14}$ s. *(7)*