Maths with Matt

Physics and maths resources

OCR AS Level Further Mathematics A

H235/Y533 - Mechanics

Friday 7 June 2024

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Friday 7 June 2024 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A particle P of mass 2.5 kg is moving with a constant speed of 4 m s^{-1} in a straight line on a smooth horizontal plane when it collides directly with a fixed vertical wall. After the collision P moves away from the wall with a speed of 2.8 m s^{-1}.
(a) Calculate the coefficient of restitution between P and the wall.
(b) Find the magnitude and state the direction of the impulse exerted on P by the wall.
(c) State the magnitude and direction of the impulse exerted on the wall by P.

Model solution

(a) The wall is fixed, so restitution is speed of separation divided by speed of approach.

$$e = \frac{2.8}{4} = 0.7.$$

$e = 0.7$. *(1)*

(b) Take the initial direction of P, towards the wall, as positive. Impulse is change in momentum.

$$I = \Delta(mv) = 2.5(-2.8) - 2.5(4) = -17 \text{ N s}.$$

The negative sign shows that the impulse is away from the wall.

The impulse on P has magnitude 17 N s, away from the wall. *(3)*

(c) By Newton's third law, the impulse on the wall is equal and opposite.

The impulse on the wall has magnitude 17 N s, towards the wall. *(2)*

Question 2

Question 2

A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 1.8 m. The other end of the string is attached to a fixed point, O, on a smooth horizontal plane. Initially, P is moving with a constant speed of 12 m s^{-1} in a horizontal circle with O as its centre.

(a) (i) Find the magnitude of the acceleration of P.

(ii) State the direction of the acceleration of P.

A force is now applied to P in such a way that its angular velocity increases. At the instant that the angular velocity reaches 8 rad s^{-1}, the string breaks.

(b) (i) Find the speed with which P is moving at the instant that the string breaks.

(ii) Find the tension in the string at the instant that the string breaks.

After the string has broken P starts to move directly up a smooth slope which is fixed to the plane and inclined at an angle $i°$ above the horizontal. Particle P moves a distance of 20 m up the slope before coming to instantaneous rest.

(c) Use an energy method to determine the value of i.

Model solution

(a)(i) Circular motion has radial acceleration v^2/r.

$$a = \frac{12^2}{1.8} = 80 \text{ m s}^{-2}.$$

80 **m s^{-2}.** **(1)**

(a)(ii) Centripetal acceleration points to the centre of the circle.

Towards O. **(1)**

(b)(i) Use $v = r\omega$.

$$v = 1.8(8) = 14.4 \text{ m s}^{-1}.$$

14.4 **m s^{-1}.** **(1)**

(b)(ii) Tension is the only horizontal radial force, so it provides the centripetal force.

$$T = mr\omega^2 = 0.4(1.8)(8^2) = 46.08 \text{ N}.$$

46.1 **N.** **(2)**

(c) The smooth slope means that the initial kinetic energy becomes gravitational potential energy. The rise is $20 \sin i$.

$$\frac{1}{2}(0.4)(14.4)^2 = 0.4(9.8)(20 \sin i),$$

$$41.472 = 78.4 \sin i, \qquad i = \sin^{-1}\left(\frac{41.472}{78.4}\right) = 31.9365\ldots°.$$

$i = 31.9°$ **(3 s.f.).** **(3)**

Question 3

Question 3

A small object P of mass m is suspended from a fixed point by a light inextensible string of length l. When P is displaced and released in a certain way, it oscillates in a vertical plane. The time taken for one complete oscillation is called the period and is denoted by τ.
A student is carrying out experiments with P and suggests the following formula to model the value of τ.

$$\tau = cg^{\alpha}l^{\beta}m^{\gamma}$$

in which g is the acceleration due to gravity and c is a dimensionless constant. (a) Use dimensional analysis to determine the values of the constants α, β and γ.
(b) (i) Determine the effect on the period, according to the model, if the length of the string is then multiplied by 4, all other conditions being unchanged.

(ii) Determine the effect on the period, according to the model, if instead the mass of the object is multiplied by 4, all other conditions being unchanged.

Model solution

(a) The right side must have the dimensions of time; c is dimensionless.

$$[\tau] = \mathrm{T}, \quad [g] = \mathrm{LT}^{-2}, \quad [l] = \mathrm{L}, \quad [m] = \mathrm{M},$$

$$\mathrm{T} = (\mathrm{LT}^{-2})^{\alpha}\mathrm{L}^{\beta}\mathrm{M}^{\gamma} = \mathrm{L}^{\alpha+\beta}\mathrm{T}^{-2\alpha}\mathrm{M}^{\gamma}.$$

$$\begin{array}{rrl} \mathrm{M}: & \gamma & = 0, \\ \mathrm{L}: & \alpha+\beta & = 0, \\ \mathrm{T}: & -2\alpha & = 1. \end{array} \qquad \alpha = -\frac{1}{2}, \quad \beta = \frac{1}{2}, \quad \gamma = 0.$$

$\alpha = -\frac{1}{2},\ \beta = \frac{1}{2},\ \gamma = 0.$ *(5)*

(b)(i) The model is $\tau = c\sqrt{l/g}$, so τ is proportional to $\sqrt{l}$.

$$\frac{\tau_{\text{new}}}{\tau_{\text{old}}} = \sqrt{\frac{4l}{l}} = 2.$$

The period is doubled. *(2)*

(b)(ii) The exponent of m is zero, so the model does not depend on mass.

There is no change in the period. *(1)*

Question 4

Question 4

A particle B of mass 5 kg is at rest at the bottom of a slope which is angled at $\sin^{-1}(0.2)$ above the horizontal. A constant force D initially acts directly up the slope on B.
The total resistance to the motion of B is modelled as being a constant 12 N.
At the instant that D stops acting, the speed of B is 18 m s^{-1} and B has moved 90 m up the slope.
Determine the average power of D over the time that D has been acting on B.

Model solution

Use work--energy: the work done by D produces kinetic energy, gravitational potential energy and the energy lost against resistance.

$$W_R = 12(90) = 1080 \text{ J}, \qquad \Delta KE = \frac{1}{2}(5)(18^2) = 810 \text{ J},$$

$$\Delta PE = 5(9.8)(90)(0.2) = 882 \text{ J}, \qquad W_D = 1080 + 810 + 882 = 2772 \text{ J}.$$

The acceleration is constant, so the average speed from rest to 18 m s^{-1} is 9 m s^{-1}.

$$90 = 9t \quad \Rightarrow \quad t = 10 \text{ s}, \qquad \overline{P} = \frac{2772}{10} = 277.2 \text{ W}.$$

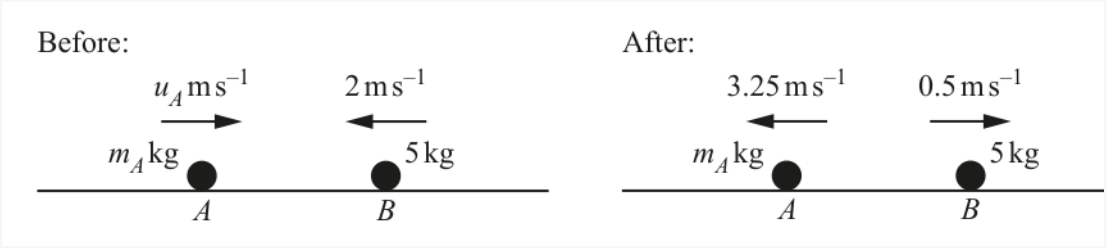
277 *W (3 s.f.).* *(6)*

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Question 5

Question 5

Two particles, A of mass m_A kg and B of mass 5 kg, are moving directly towards each other on a smooth horizontal floor. Before they collide they have speeds u_A m s^{-1} and 2 m s^{-1} respectively. Immediately after they collide the direction of motion of each particle has been reversed and A and B have speeds 3.25 m s^{-1} and 0.5 m s^{-1} respectively (see diagram). The coefficient of restitution between A and B is 0.75.



(a) Determine the value of m_A and the value of u_A.
(b) Show that approximately 41% of the kinetic energy of the system is lost in this collision.
After the collision between A and B, B goes on to collide directly with a third particle C of mass 3 kg which is travelling towards B with a speed of 5.5 m s^{-1}. The coefficient of restitution between B and C is denoted by e.
(c) Given that, after B and C collide, there are no further collisions between A, B and C, determine the range of possible values of e.

Model solution

(a) Take the initial direction of A as positive. Momentum is conserved on the smooth floor, and restitution supplies a second equation.

$$m_A u_A + 5(-2) = m_A(-3.25) + 5(0.5),$$

$$\frac{0.5 - (-3.25)}{u_A - (-2)} = 0.75.$$

$$3.75 = 0.75(u_A + 2) \Rightarrow u_A = 3,$$

$$3m_A - 10 = -3.25m_A + 2.5 \Rightarrow 6.25m_A = 12.5 \Rightarrow m_A = 2.$$

$m_A = 2$ ***kg***, $u_A = 3$ ***m s^{-1}***. *(5)*

(b) Compare total kinetic energy before and after the collision.

$$KE_{\text{before}} = \frac{1}{2}(2)(3^2) + \frac{1}{2}(5)(2^2) = 19 \text{ J},$$

$$KE_{\text{after}} = \frac{1}{2}(2)(3.25^2) + \frac{1}{2}(5)(0.5^2) = 11.1875 \text{ J},$$

$$\frac{19 - 11.1875}{19} \times 100 = 41.118\ldots\%.$$

Approximately 41% of the kinetic energy is lost. *(3)*

(c) Let the velocities after the collision of B and C be V_B and V_C, retaining the positive direction from part (a).

$$5(0.5) + 3(-5.5) = 5V_B + 3V_C, \qquad 5V_B + 3V_C = -14,$$

$$\frac{V_C - V_B}{0.5 - (-5.5)} = e \Rightarrow V_C - V_B = 6e.$$

$$3(V_C - V_B) = 18e, \quad 8V_B = -14 - 18e, \quad V_B = -\frac{7}{4} - \frac{9}{4}e.$$

Particle A has velocity -3.25 m s^{-1}. To avoid B catching it, $V_B \geq -3.25$.

$$-\frac{7}{4} - \frac{9}{4}e \geq -\frac{13}{4} \Rightarrow e \leq \frac{2}{3}.$$

Also $0 \leq e \leq 1$.

$0 \leq e \leq \frac{2}{3}$. **(6)**

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Question 6

Question 6

A motorbike and its rider, together denoted by M, have a combined mass of 360 kg. The resistive force experienced by M when it is in motion is modelled as being proportional to the speed it is moving at. All motion of M is on a straight horizontal road.
It is found that with the engine of the motorbike working at a rate of 12 kW, the maximum constant speed that M can move at is 10 m s^{-1}.
Determine the speed of M such that with the engine working at a rate of 12 kW the acceleration of M is 1.5 m s^{-2}.

Model solution

Let resistance be kv. At the maximum constant speed, the resultant force is zero. Use $P = Dv$ to find the driving force.

$$12000 = D(10) \Rightarrow D = 1200 \text{ N}, \qquad 1200 = k(10) \Rightarrow k = 120.$$

At the required speed, the driving force is $12000/v$, since power remains 12000 W. Apply Newton's second law.

$$\frac{12000}{v} - 120v = 360(1.5),$$

$$12000 - 120v^2 = 540v, \qquad 2v^2 + 9v - 200 = 0,$$

$$(2v + 25)(v - 8) = 0, \qquad v = 8 \text{ or } v = -12.5.$$

Speed is positive, so reject the negative root.

8 *m s^{-1}.* *(7)*

Question 7

Question 7

A particle P of mass 3.5 kg is attached to one end of a rod of length 5.4 m. The other end of the rod is hinged at a fixed point O and P hangs in equilibrium directly below O.
A horizontal impulse of magnitude 44.1 N s is applied to P.
In an initial model of the subsequent motion of P the rod is modelled as being light and inextensible and all resistance to the motion of P is ignored. You are given that P moves in a circular path in a vertical plane containing O. The angle that the rod makes with the downward vertical through O is θ radians.
(a) Determine the largest value of θ in the subsequent motion of P.
In a revised model the rod is still modelled as being light and inextensible but the resistance to the motion of P is not ignored. Instead, it is modelled as causing a loss of energy of 20 J for every metre that P travels.
(b) Show that according to the revised model, the maximum value of θ in the subsequent motion of P satisfies the following equation.

$$343(1 + 2\cos\theta) = 400\theta$$

You are given that $\theta = 1.306$ is the solution to the above equation, correct to 4 significant figures.
(c) Determine the difference in the predicted maximum vertical heights attained by P using the two models. Give your answer correct to 3 significant figures.
(d) Suggest one further improvement that could be made to the model of the motion of P.

Model solution

(a) First use impulse--momentum to find the initial speed. With no resistance, energy is conserved until the particle is instantaneously at rest at its greatest height.

$$44.1 = 3.5u \Rightarrow u = 12.6 \text{ m s}^{-1}, \qquad KE = \frac{1}{2}(3.5)(12.6)^2 = 277.83 \text{ J}.$$

At angle θ, the vertical rise is $5.4(1 - \cos\theta)$.

$$277.83 = 3.5(9.8)\left[5.4(1 - \cos\theta)\right] = 185.22(1 - \cos\theta),$$

$$\cos\theta = -\frac{1}{2}.$$

The first maximum is in the interval $0 < \theta < \pi$.

$$\theta = \frac{2\pi}{3}.$$

$\theta = \dfrac{2\pi}{3}$ *rad.* **(5)**

(b) The resistance removes 20 J per metre, and the arc length travelled is 5.4θ. Apply work--energy.

$$277.83 = 3.5(9.8)\left[5.4(1 - \cos\theta)\right] + 20(5.4\theta),$$

$$277.83 = 185.22 - 185.22\cos\theta + 108\theta,$$

$$343 + 686\cos\theta = 400\theta, \qquad \boxed{343(1 + 2\cos\theta) = 400\theta}.$$

$343(1 + 2\cos\theta) = 400\theta$, *as required.* **(2)**

(c) The two maximum heights, measured from the lowest point, are

$$h_1 = 5.4\left(1 - \cos\frac{2\pi}{3}\right) = 8.1 \text{ m},$$

$$h_2 = 5.4(1 - \cos 1.306) = 3.986\ldots \text{ m}.$$

$$h_1 - h_2 = 8.1 - 3.986\ldots = 4.113\ldots \text{ m}.$$

4.11 *m (3 s.f.).* *(3)*

(d) The rod being light is an idealisation; its mass affects the energy required to move it.

Include the mass (and weight) of the rod. *(1)*

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