Maths with Matt

Physics and maths resources

OCR AS Level Further Mathematics A

H235/Y533 - Mechanics

Tuesday 20 May 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Tuesday 20 May 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A cricket ball of mass 0.16 kg is at rest on rough horizontal ground when it is struck by a bat. Immediately after the ball is struck its speed is 15 m s^{-1} in a horizontal direction.
(**a**) Find the magnitude of the impulse exerted by the bat on the ball. [1]
The ball remains in contact with the ground and the resistance to the motion of the ball is modelled as constant. The ball comes to rest again after it has travelled 30 m along the ground.
(**b**) Use the work-energy principle to find the magnitude of the resistance to the motion of the ball. [3]

Model solution

(a) Impulse is the change in momentum. The ball starts from rest, so its final momentum gives the impulse magnitude.

$$I = \Delta(mv) = 0.16(15) - 0 = 2.4 \text{ N s}.$$

The impulse has magnitude 2.4 *N s.* *([1])*

(b) The only horizontal force doing work after the strike is the constant resistance. The initial kinetic energy is all removed by this negative work.

$$\text{initial KE} = \frac{1}{2}(0.16)(15)^2 = 18 \text{ J}.$$

$$-R(30) = 0 - 18, \qquad 30R = 18, \qquad R = 0.6 \text{ N}.$$

The magnitude of the resistance is 0.600 *N.* *([3])*

Question 2

Question 2

Two particles, A of mass m kg and B of mass $2m$ kg, are connected by a light inextensible string. The particles are initially placed together at rest on a smooth horizontal surface with the string slack.
At a certain instant, A and B are projected along the surface, directly away from each other, with speeds 3 m s^{-1} and u m s^{-1} respectively. You are given that $u < 1.5$.
(a) Find, in terms of u, the speed of A immediately after the string becomes taut. [2]
(b)(i) State the direction of motion of B immediately after the string becomes taut. [1]
(ii) Find, in terms of m and u, the magnitude of the impulse exerted on B as a result of the string becoming taut. [2]
(iii) Write down an expression for the impulse exerted on A as a result of the string becoming taut. [1]
(c) Show that the loss of kinetic energy as a result of the string becoming taut is $\frac{m(3+u)^2}{3}$ J. [3]

Model solution

Take A's original direction as positive. When the string becomes taut, its inextensibility makes the two particles have one common velocity v. There is no external horizontal impulse, so momentum is conserved.

$$3m + 2m(-u) = (m + 2m)v.$$

$$v = \frac{3 - 2u}{3}.$$

Since $u < 1.5$, $3 - 2u > 0$, so the particles move in A's original direction; the speed of A is therefore $\frac{3-2u}{3}$ m s^{-1}.

(a) The speed of A is $\frac{3-2u}{3}$ m s^{-1}. *([2])*

Both particles have the common positive velocity found above.

(b)(i) B moves towards A (opposite to B's original direction). *([1])*

(b)(ii) Use impulse as change in momentum of B, retaining the chosen positive direction.

$$J_B = 2m\left(\frac{3-2u}{3} - (-u)\right) = \frac{2m(3+u)}{3} \text{ N s.}$$

(b)(ii) The impulse on B has magnitude $\frac{2m(3+u)}{3}$ N s. *([2])*

The impulses caused by the taut string form an equal and opposite pair.

(b)(iii) The impulse on A is $-\frac{2m(3+u)}{3}$ N s, in A's original positive direction convention. *([1])*

(c) Compare the total kinetic energy before and after the impulsive event. The common speed must

be used for both particles afterwards.

$$E_{\text{before}} = \frac{1}{2}m(3)^2 + \frac{1}{2}(2m)u^2 = \frac{9m}{2} + mu^2,$$

$$E_{\text{after}} = \frac{1}{2}(3m)\left(\frac{3-2u}{3}\right)^2 = \frac{m(3-2u)^2}{6}.$$

$$\begin{aligned} E_{\text{before}} - E_{\text{after}} &= \frac{9m}{2} + mu^2 - \frac{m(3-2u)^2}{6} \\ &= \frac{m}{6}\left(27 + 6u^2 - 9 + 12u - 4u^2\right) \\ &= \frac{m(3+u)^2}{3} \text{ J}. \end{aligned}$$

(c) The loss of kinetic energy is $\frac{m(3+u)^2}{3}$ J, as required. *([3])*

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Question 3

Question 3

A car of mass 900 kg has an engine that is capable of working at a maximum power of 12 kW. When the car is travelling along a straight horizontal road, the maximum constant speed that it can maintain is 40 m s^{-1}.
In an initial model of the motion of the car it is assumed that the total resistance to motion is constant.
(a)(i) Show that when the car is travelling at 15 m s^{-1} along the horizontal road with the engine working at a constant rate of 10 kW, the acceleration of the car is 0.407 m s^{-2} correct to 3 significant figures. [3]
(ii) Find the speed of the car when it is accelerating at 0.2 m s^{-2} along the horizontal road with the engine working at a constant rate of 9.6 kW. [2]
(iii) The car now starts to descend a straight road which is inclined at $1°$ to the horizontal. Find the power produced by the engine of the car when it is travelling along this road at a constant speed of 40 m s^{-1}. [2]
In a refined model of the motion of the car it is assumed that the total resistance to motion is proportional to the speed of the car.
(b)(i) Show that the total resistance to the motion of the car is now given by $7.5v$ N, where v m s^{-1} is the speed of the car. [1]
(ii) State one way in which the refined model is an improvement over the initial model. [1]
(iii) Find, according to the refined model, the speed of the car when it is accelerating at 0.2 m s^{-2} along the horizontal road with the engine working at a constant rate of 9.6 kW. [3]

Model solution

At constant speed the resultant force is zero. Since power equals driving force times speed, the maximum-speed information first fixes the resistance in the initial model.

$$R = \frac{12000}{40} = 300 \text{ N}.$$

(a)(i) At 15 m s^{-1}, the driving force is P/v. Apply Newton's second law in the direction of travel.

$$\frac{10000}{15} - 300 = 900a, \qquad a = \frac{11}{27} = 0.407\ldots \text{ m s}^{-2}.$$

(a)(i) $a = 0.407$ m s^{-2} to 3 significant figures. *([3])*

(a)(ii) The same force balance applies, now with the specified acceleration.

$$\frac{9600}{v} - 300 = 900(0.2) = 180, \qquad \frac{9600}{v} = 480, \qquad v = 20.$$

(a)(ii) The speed is 20.0 m s^{-1}. *([2])*

(a)(iii) Take downhill as positive. At constant speed, the downhill component of weight and the engine driving force balance the 300 N resistance.

$$\frac{P}{40} + 900(9.8)\sin 1° - 300 = 0.$$

$$P = 40\left(300 - 900(9.8)\sin 1°\right) = 5842.8\ldots \text{ W}.$$

(a)(iii) The engine power is 5.84 kW. *([2])*

(b)(i) Write the refined resistance as $R = kv$, then use the same maximum constant-speed condition.

$$300 = k(40), \qquad k = 7.5, \qquad R = 7.5v \text{ N}.$$

(b)(i) $R = 7.5v$ N. *([1])*

(b)(ii) Resistance increasing with speed is more realistic, especially for air resistance, than assuming it is constant. *([1])*

(b)(iii) Use $R = 7.5v$ in Newton's second law, while the driving force remains P/v.

$$\frac{9600}{v} - 7.5v = 900(0.2) = 180.$$

$$7.5v^2 + 180v - 9600 = 0, \qquad v^2 + 24v - 1280 = 0.$$

$$v = \frac{-24 \pm \sqrt{24^2 + 4(1280)}}{2} = -12 \pm 4\sqrt{89}.$$

A speed cannot be negative, so reject the negative root.

(b)(iii) The speed is $-12 + 4\sqrt{89} = 25.7$ m s^{-1} to 3 significant figures. *([3])*

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Question 4

Question 4

Two particles, A of mass 4 kg and B of mass 2 kg, are free to move along the same straight line on a smooth horizontal surface. They are projected directly towards each other with speeds 6 m s^{-1} and 5 m s^{-1} respectively.
After A and B collide the speed of A is 1 m s^{-1}.
(**a**) Determine the possible values of the coefficient of restitution between A and B. [6]
(**b**) State whether the collision between A and B is perfectly elastic. Justify your answer. [1]

Model solution

Take A's initial direction as positive, so initially $u_A = 6$ and $u_B = -5$. A's stated speed after collision permits two directions, so both cases must be considered. Momentum is conserved because the surface is smooth.

$$4(6) + 2(-5) = 14.$$

If A continues in its original direction, $v_A = 1$.

$$14 = 4(1) + 2v_B, \qquad v_B = 5.$$

$$e = \frac{\textit{speed of separation}}{\textit{speed of approach}} = \frac{5-1}{6-(-5)} = \frac{4}{11}.$$

If A reverses direction, $v_A = -1$.

$$14 = 4(-1) + 2v_B, \qquad v_B = 9,$$

$$e = \frac{9-(-1)}{6-(-5)} = \frac{10}{11}.$$

(a) The possible coefficients of restitution are $\frac{4}{11}$ and $\frac{10}{11}$. ([6])

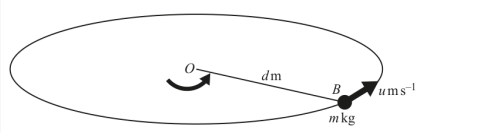
A perfectly elastic collision has coefficient of restitution $e = 1$. Neither possible value has this value.

(b) No; the collision is not perfectly elastic because $e \neq 1$ in either case. ([1])

Question 5

Question 5

One end of a light inextensible string of length d m is attached to a particle B of mass m kg. The other end of the string is attached to a fixed point O on a smooth horizontal plane. Particle B moves on the plane with constant speed u m s^{-1} in a horizontal circle with centre O (see diagram).



The period, which is the time taken for B to complete one revolution, is denoted by p seconds and the tension in the string is F N.
(a) Use dimensional analysis to derive a formula for F in terms of d, m and p. You should define any other symbols used in your formula for F. [4]
(b) Explain why it was not necessary to consider the quantity u in the analysis in part (a). [1]
(c) Instead, use Newton's second law to derive a formula for F in terms of d, m and p. [3]
(d) Considering your answers to part (a) and part (c), state one disadvantage of using dimensional analysis to derive a formula for F. [1]

Model solution

(a) Assume a product of powers of the relevant variables. A dimensionless constant is needed because dimensions alone cannot fix a numerical multiplier.

$$F = kd^{\alpha}m^{\beta}p^{\gamma},$$

where k is a dimensionless constant. Equating the dimensions on both sides gives

$$[F] = MLT^{-2} = [d]^{\alpha}[m]^{\beta}[p]^{\gamma} = L^{\alpha}M^{\beta}T^{\gamma}.$$

$$\alpha = 1, \qquad \beta = 1, \qquad \gamma = -2.$$

(a) $F = kdmp^{-2} = \dfrac{kdm}{p^2}$, *where k is dimensionless.* *([4])*

(b) The speed is already determined by the radius and the period: in one period B travels one circumference.

$$u = \frac{2\pi d}{p}.$$

(b) It is unnecessary to include u, because $u = \dfrac{2\pi d}{p}$ *is determined by d and p.* *([1])*

(c) Tension is the resultant force towards the centre, so Newton's second law is applied to the centripetal acceleration.

$$F = m\frac{u^2}{d}.$$

$$u = \frac{2\pi d}{p} \quad \Longrightarrow \quad F = m\frac{\left(\frac{2\pi d}{p}\right)^2}{d} = \frac{4\pi^2 dm}{p^2}.$$

(c) $F = \dfrac{4\pi^2 dm}{p^2}$. *([3])*

(d) Dimensional analysis cannot determine the numerical value of the dimensionless constant k. *([1])*

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Question 6

Question 6

A disc of mass 2 kg is projected, with initial speed 16.4 m s^{-1}, directly up a rough plane which is inclined at an angle of $15°$ to the horizontal. The coefficient of friction between the plane and the disc is 0.2.
(a)(i) Determine the average power generated by the frictional force in the first 3 seconds of the disc's motion. [6]
(ii) Explain the significance of the sign of your answer to part (a)(i). [1]
(b) Use an energy method to find the distance the disc travels up the plane from the point where it is projected to the point where it comes to instantaneous rest. [4]

Model solution

(a)(i) Take up the plane as positive. While the disc moves up the plane, both friction and the component of its weight act down the plane. First find the friction from the normal reaction.

$$R = 2(9.8)\cos 15° = 18.932\ldots \text{ N},$$

$$F_{\text{fr}} = 0.2R = 3.7864\ldots \text{ N}.$$

$$2a = -2(9.8)\sin 15° - 3.7864\ldots, \qquad a = -4.4296\ldots \text{ m s}^{-2}.$$

The speed after three seconds is still positive, so this calculation applies throughout the stated interval. The distance travelled in that interval is

$$s = 16.4(3) + \frac{1}{2}(-4.4296\ldots)(3)^2 = 29.266\ldots \text{ m}.$$

Friction does negative work because its direction is opposite to the displacement. Average power is work done divided by time.

$$\overline{P} = \frac{-F_{\text{fr}}s}{3} = \frac{-(3.7864\ldots)(29.266\ldots)}{3} = -36.9\ldots \text{ W}.$$

(a)(i) The average power generated by friction is -36.9 W. *([6])*

(a)(ii) The negative sign shows that friction removes mechanical energy from the disc; work is being done against friction. *([1])*

(b) An energy equation directly relates the initial kinetic energy to the gain in gravitational potential energy and the work done against friction. Let d be the distance up the plane until rest.

$$\frac{1}{2}(2)(16.4)^2 = 2(9.8)d\sin 15° + (0.2 \times 2(9.8)\cos 15°)\,d.$$

$$268.96 = (2(9.8)\sin 15° + 0.2 \times 2(9.8)\cos 15°)\,d,$$

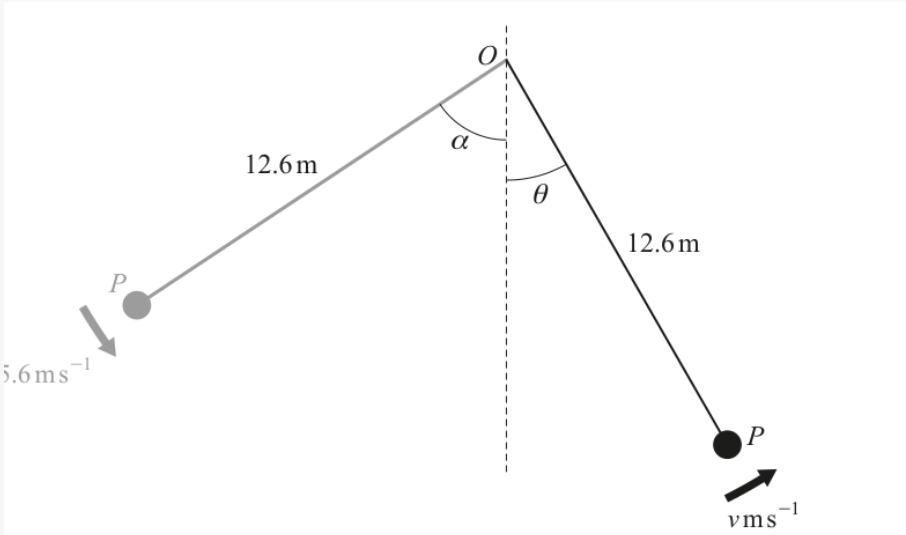
$$d = 30.359\ldots \text{ m}.$$

(b) The disc travels 30.4 m up the plane. *([4])*

Question 7

Question 7

In this question you must show detailed reasoning.
A particle P of mass m kg is attached to one end of a light rod of length 12.6 m. The other end of the rod is attached to a fixed point O.
Particle P is initially held at a point so that OP makes an angle of α to the downward vertical through O, where $\cos\alpha = \frac{5}{9}$.
Particle P is then projected so that it instantaneously starts moving downwards at right angles to the rod with a speed of 5.6 m s^{-1} in a vertical circular path around O.
When the rod makes an angle of θ with the downward vertical, P has speed v m s^{-1} (see diagram).



(a) At a certain instant in P's motion, $v = 8.4\cos\theta$.

Determine the value of v at this instant.

(b) Show that P does **not** describe complete circles around O.

(**a**) At a certain instant in P's motion, $v = 8.4\cos\theta$. Determine the value of v at this instant. [6]
(**b**) Show that P does not describe complete circles around O. [2]

Model solution

(a) Use conservation of mechanical energy, taking the lowest point of the circle as the zero for gravitational potential energy. Initially the height above this point is $12.6 - 12.6\cos\alpha$.

$$E_{\text{initial}} = \frac{1}{2}m(5.6)^2 + mg(12.6 - 12.6\cos\alpha).$$

$$E_{\text{initial}} = 15.68m + 9.8m\left(12.6 - 12.6\cdot\frac{5}{9}\right) = 15.68m + 54.88m = 70.56m.$$

At the stated instant, write the kinetic and potential energies in terms of θ, then substitute the given relation between speed and angle.

$$\frac{1}{2}mv^2 + mg(12.6 - 12.6\cos\theta) = 70.56m,$$

$$\frac{1}{2}m(8.4\cos\theta)^2 + 9.8m(12.6 - 12.6\cos\theta) = 70.56m.$$

$$35.28\cos^2\theta - 123.48\cos\theta + 52.92 = 0.$$

$$2\cos^2\theta - 7\cos\theta + 3 = 0, \qquad (2\cos\theta - 1)(\cos\theta - 3) = 0.$$

The root $\cos\theta = 3$ is impossible because $-1 \leq \cos\theta \leq 1$. Thus $\cos\theta = \frac{1}{2}$.

$$v = 8.4\left(\frac{1}{2}\right) = 4.2 \text{ m s}^{-1}.$$

(a) $v = 4.2$ m s^{-1}. ([6])

(b) A complete circle would require P to reach the top, which is $2(12.6)$ m above the chosen zero level. Compare the available energy with the potential energy needed even if the speed at the top were zero.

$$E_{\text{top}} = mg(2 \times 12.6) = 246.96m \text{ J}.$$

$$E_{\text{initial}} = 70.56m \text{ J} < 246.96m \text{ J} = E_{\text{top}}.$$

(b) P does not have enough energy to reach the top of the circle, so it cannot describe complete circles around O. ([2])

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