Maths with Matt

Physics and maths resources

OCR AS Level Further Mathematics A

H235/Y534 - Discrete Mathematics

Tuesday 3 June 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the OCR Tuesday 3 June 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A builder has some pieces of wood that are 6 m in length. The builder needs to cut smaller pieces to use in a project. The lengths of the smaller pieces required are shown below.

$$2.5\text{ m} \quad 2\text{ m} \quad 4.5\text{ m} \quad 3\text{ m} \quad 1.5\text{ m} \quad 3.5\text{ m} \quad 2\text{ m}$$

(a) Use the first-fit decreasing algorithm to determine the smallest number of 6 m lengths of wood needed to cut the seven smaller pieces. [3]
(b) First-fit decreasing is a heuristic algorithm. Explain what this means. [1]
The first-fit decreasing algorithm is order $O(n^2)$.
(c) It takes a computer 3.2 seconds to solve an allocation problem involving 200 items of data using the first-fit decreasing algorithm. Calculate approximately how long it would take to solve an allocation problem involving 1000 items of data. [2]

Model solution

(a) First-fit decreasing first puts the pieces in decreasing order, then places each piece in the first length of wood in which it fits.

$$4.5,\ 3.5,\ 3,\ 2.5,\ 2,\ 2,\ 1.5$$

length of wood	*pieces placed*	*total length*
1	4.5, 1.5	6
2	3.5, 2.5	6
3	3, 2	5
4	2	2

The total required length is 19 m, which is greater than $3 \times 6 = 18$ m, so three lengths cannot suffice.

The smallest number needed is 4 lengths of 6 m. **(3)**

(b) A heuristic is appropriate when a good answer is needed efficiently, even though the method is not guaranteed to find the optimum.

It is a method that finds a valid solution quickly, but may not find the optimal solution. **(1)**

(c) Since the running time is proportional to n^2, increasing the number of items from 200 to 1000 multiplies the time by $\left(\frac{1000}{200}\right)^2$.

$$3.2\left(\frac{1000}{200}\right)^2 = 3.2(5)^2 = 80.$$

Approximately 80 seconds, or 1 minute 20 seconds. **(2)**

Question 2

Question 2

A student has six cards numbered from 1–6.
(a) The student chooses four cards at random. Use the Pigeonhole Principle to explain why they must have at least two cards whose numbers add to seven. [2]
(b) The student now chooses two cards and places them side by side to form a two-digit number.
(i) Determine the number of different two-digit numbers that the student can make. [2]
(ii) Determine the number of different even two-digit numbers that the student can make. [2]

Model solution

(a) Pair the cards into the three pairs whose values sum to seven. These are the pigeonholes.

$$(1,6), \qquad (2,5), \qquad (3,4).$$

Four cards are chosen but there are only three pairs. By the Pigeonhole Principle, two chosen cards must be in the same pair, so their numbers add to 7.

At least two of the four cards have numbers that add to 7. **(2)**

(b)(i) The tens digit can be any of the six cards. Once it is used, five cards remain for the units digit.

$$6 \times 5 = 30.$$

30 *different two-digit numbers.* **(2)**

(b)(ii) An even number must have an even units digit. There are three choices, 2, 4, 6, and then five remaining choices for the tens digit.

$$3 \times 5 = 15.$$

15 *different even two-digit numbers.* **(2)**

Question 3

Question 3

The table shows the lengths in km of footpaths between six places, A to F, in a town.

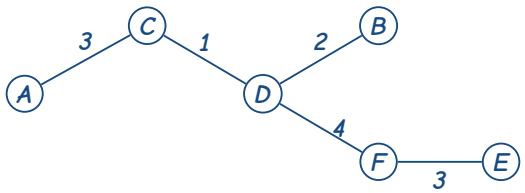
	A	B	C	D	E	F
A	–	5	3	5	8	10
B	5	–	6	2	7	5
C	3	6	–	1	10	8
D	5	2	1	–	9	4
E	8	7	10	9	–	3
F	10	5	8	4	3	–

(a) Apply the tabular form of Prim's algorithm, starting at A, to the copy of the table in the Printed Answer Booklet to construct a minimum spanning tree for the six places in the town. You should state the order in which the arcs are added to the tree.
Draw the minimum spanning tree and find its weight. [4]
(b) Sam needs to walk from A to F. Give an example of a question Sam could ask which leads to an enumeration problem. [1]
(c) Use an appropriate algorithm to find the shortest route from A to F. You must state its length. [4]

Model solution

(a) Prim's algorithm grows one connected tree. Starting at A, at each stage choose the shortest arc from the current tree to a new vertex.

current tree	*shortest eligible arc*	*new tree*
A	AC (3)	A, C
A, C	CD (1)	A, C, D
A, C, D	DB (2)	A, C, D, B
A, C, D, B	DF (4)	A, C, D, B, F
A, C, D, B, F	FE (3)	A, C, D, B, F, E



Arcs in order: AC, CD, DB, DF, FE. *The minimum spanning-tree weight is* $3 + 1 + 2 + 4 + 3 = 13$ *km.* *(4)*

(b) Enumeration means counting the possible objects, rather than optimising a route.

For example: "How many different walks are there from A to F?" *(1)*

(c) All arc lengths are non-negative, so Dijkstra's algorithm is suitable. The table records the

permanent distance from A; tentative distances are updated through each newly permanent vertex.

permanent vertex	*permanent distance*	*predecessor*	*useful updated tentative distances*
A	0	$-$	$B=5,\ C=3,\ D=5,\ E=8,\ F=10$
C	3	A	$D=4,\ F=11$
D	4	C	$B=5,\ F=8,\ E=13$
B	5	A	$E=12,\ F=10$
F	8	D	*stop*

Tracing predecessors backwards from F gives $F \leftarrow D \leftarrow C \leftarrow A$.

Shortest route: $A \rightarrow C \rightarrow D \rightarrow F$, *of length* 8 *km.* **(4)**

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Question 4

Question 4

Ryan and Casey play a game. They can each choose to play one of three options. Ryan can play options A, B and C and Casey can play options X, Y and Z. The scores for each player for each combination of choices are shown in the pay-off matrices below.
Scores for Ryan:

Ryan's options\Casey's options	X	Y	Z
A	1	4	3
B	5	3	4
C	3	1	3

Scores for Casey:

Ryan's options\Casey's options	X	Y	Z
A	5	2	3
B	1	3	2
C	3	5	3

(a) Explain how the game between Ryan and Casey can be expressed as the zero-sum game with the pay-off matrix below (the entries represent pay-offs for Ryan). [1]

Ryan's options\Casey's options	X	Y	Z
A	-2	1	0
B	2	0	1
C	0	-2	0

(b) If Casey knows that Ryan will play option A, state the option that Casey should play. [1]
(c) Find the play-safe strategies for both players. [3]
(d) Determine whether or not the game is stable. [1]
(e) Use a dominance argument to reduce the zero-sum game to a 2×3 game. [1]
(f) Let p be the probability that Ryan plays option A on the reduced game from part (e). Use a graphical method to determine the optimal mixed strategy for Ryan. [4]

Model solution

(a) In every outcome the two original scores add to the same total, 6. Subtracting 3 from Ryan's score therefore gives the opposite of Casey's adjusted score, making the total zero.

$$1 - 3 = -2, \qquad 4 - 3 = 1, \qquad \textit{and similarly for every entry.}$$

Subtract 3 from every one of Ryan's scores; since the two scores always total 6, this gives the stated zero-sum pay-offs. **(1)**

(b) Casey minimises Ryan's pay-off. In row A the pay-offs are $-2, 1, 0$, whose smallest value is under X.

X. **(1)**

(c) Ryan's play-safe choice maximises the minimum in a row. Casey's play-safe choices minimise the maximum in a column.

	X	Y	Z	*row minimum*
A	-2	1	0	-2
B	2	0	1	0
C	0	-2	0	-2

	X	Y	Z
column maximum	2	1	1

Ryan's play-safe strategy is B. Casey's play-safe strategies are Y and Z. **(3)**

(d) A stable game has equal maximin and minimax values. Here these are 0 and 1.

The game is not stable, since $0 \neq 1$. **(1)**

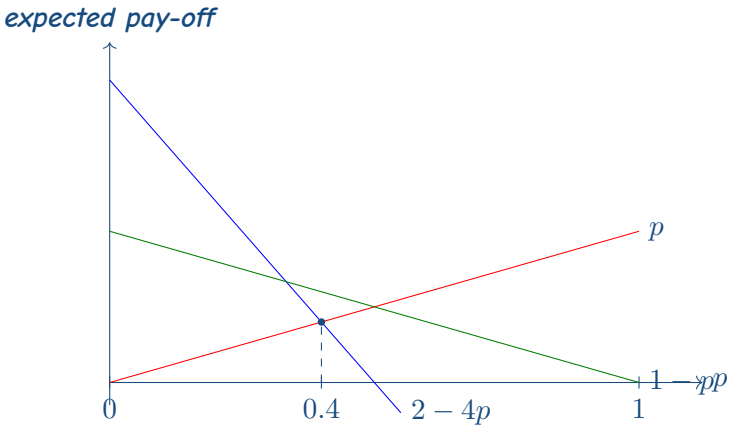
(e) Ryan prefers the larger pay-off. Each entry in row B exceeds the corresponding entry in row C, so B strictly dominates C.

$$(2, 0, 1) > (0, -2, 0).$$

Remove row C. The reduced 2×3 game has rows A and B. **(1)**

(f) If Ryan plays A with probability p, Ryan plays B with probability $1 - p$. For each pure choice by Casey, calculate Ryan's expected pay-off; Ryan maximises the lowest of these three lines.

Casey's choice	expected pay-off for Ryan
X	$-2p + 2(1-p) = 2 - 4p$
Y	$p + 0(1-p) = p$
Z	$0p + 1(1-p) = 1 - p$



The lower envelope is greatest where $2 - 4p$ and p meet.

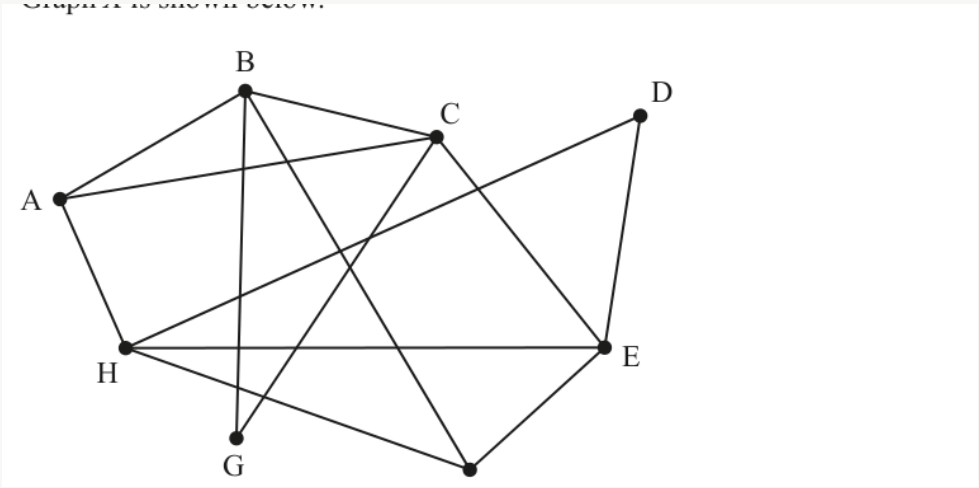
$$p = 2 - 4p \quad \Rightarrow \quad 5p = 2 \quad \Rightarrow \quad p = \frac{2}{5}.$$

Ryan plays A with probability $\frac{2}{5}$, B with probability $\frac{3}{5}$, and C with probability 0. **(4)**

Question 5

Question 5

Graph X is shown below.



(a) Explain whether graph X is Eulerian, semi-Eulerian or neither. [1]
(b) Give an example of a cycle on the graph above. [1]
The adjacency matrix for a graph Y is shown below.

	I	J	K	L	M	N	O	P
I	0	1	1	1	0	0	0	1
J	1	0	1	1	1	0	0	0
K	1	1	0	1	0	0	0	0
L	1	1	1	0	1	0	0	0
M	0	1	0	1	0	0	0	1
N	0	0	0	0	0	0	1	1
O	0	0	0	0	0	1	0	1
P	1	0	0	0	1	1	1	0

(c) Show that X and Y are not isomorphic. [3]

Model solution

(a) An Eulerian graph has no odd-degree vertices; a semi-Eulerian graph has exactly two. Counting the arcs at each vertex shows that only A and F have odd degree.

Graph X is semi-Eulerian, because A and F are the only vertices of odd degree. **(1)**

(b) A cycle starts and finishes at the same vertex without repeating an arc or an intermediate vertex.

For example, $A \to B \to C \to A$. **(1)**

(c) Isomorphisms preserve both degrees and adjacencies. In X the two degree-2 vertices are D and G, and they are not adjacent. From the row sums of the adjacency matrix, Y has two degree-2 vertices, N and O, but these are adjacent because the (N, O) *entry is* 1.

Since the relationship between the two degree-2 vertices is different, X and Y are not isomorphic. *(3)*

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Question 6

Question 6

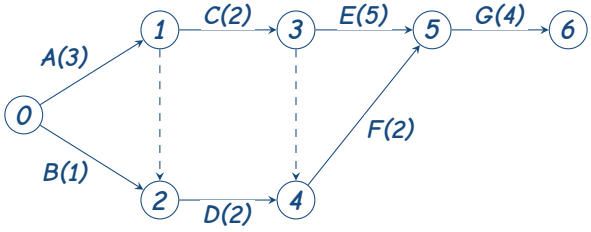
The table shows the activities involved in a project, together with their immediate predecessors and the duration of each activity in hours.

Activity	Duration (hours)	Immediate predecessors
A	3	–
B	1	–
C	2	A
D	2	A, B
E	5	C
F	2	C, D
G	4	E, F

(a) Model the project using an activity network. [3]
(b) (i) Carry out a forward pass to find the early event times and a backward pass to find the late event times. [3]
(ii) Write down the critical activities. [1]
(c) Determine the effect on the minimum project completion time if the duration of activity F is 4 hours longer than planned. [2]

Model solution

(a) An activity-on-arc network is needed. The two dashed arcs are dummy activities: they impose the required precedence without taking time.



The activity-on-arc network above, with two dummy arcs, models all the stated immediate predecessors. **(3)**

(b)(i) In the forward pass, each event time is the largest early completion time into that event. In the backward pass, each late time is the smallest late start time out of it.

event	0	1	2	3	4	5	6
early event time	0	3	3	5	5	10	14
late event time	0	3	6	5	8	10	14

The early and late event times are as shown in the table. **(3)**

(b)(ii) Critical activities have zero float: their start and finish events have equal early and late times along the activity.

A, C, E and G. **(1)**

(c) The float of F is its latest start minus earliest start.

$$\textit{float of F} = 10 - 5 - 2 = 3 \textit{ hours.}$$

Increasing F by 4 hours exceeds its float by 1 hour, so the completion time must increase by 1 hour.

The minimum completion time increases from 14 hours to 15 hours. **(2)**

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Question 7

Question 7

A shop sells three different arrangements of flowers, X, Y and Z. The number of roses, lilies and carnations in each arrangement are shown in the table below.

Arrangement	Roses	Lilies	Carnations
X	5	3	5
Y	4	7	2
Z	9	6	7

Arrangement X sells for $\$20$, arrangement Y sells for $\$15$ and arrangement Z sells for $\$24$.
The shopkeeper has a maximum of 40 roses, 80 lilies and 30 carnations that they can use to make the flower arrangements. The shopkeeper wants to make as much money as possible from selling the flower arrangements.
The shopkeeper decides that no more than 50% of the total number of arrangements can be arrangement Y.
(a) Formulate a linear programming problem to represent the problem. You should define the variables that you are using in your formulation. [5]
The shopkeeper decides to make the same number of arrangement Z as arrangement X.
(b) (i) Use a graphical method to determine how many of each arrangement should be made. [6]
(ii) State the maximum amount of money that the shopkeeper can make. [1]
The maximum number of lilies available changes from 80 to k. The number of arrangement Z is still the same as the number of arrangement X.
(c) Determine the range of values of k for which the new optimal solution is the same as in part (b)(i). [2]

Model solution

(a) Let the variables count arrangements. Each flower stock gives an upper-bound constraint, while the selling prices give the objective function.

$x =$ *number of arrangements X*,
$y =$ *number of arrangements Y*,
$z =$ *number of arrangements Z*.

Maximise $P = 20x + 15y + 24z$

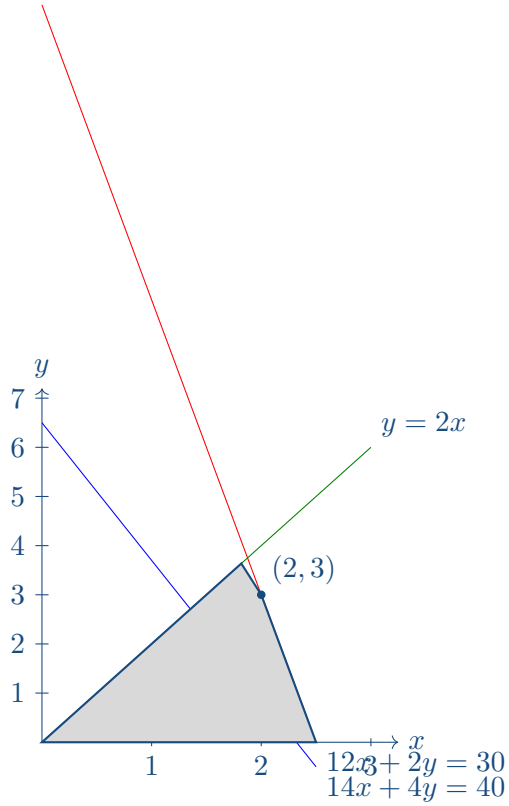
$5x + 4y + 9z \leq 40$ *(roses)*,
$3x + 7y + 6z \leq 80$ *(lilies)*,
$5x + 2y + 7z \leq 30$ *(carnations)*,
$y \leq x + z$ *(at most* 50% *are Y)*,
$x, y, z \geq 0$ *and are integers*.

The linear programme is the objective and constraints above. *(5)*

(b)(i) Substitute $z = x$, reducing the problem to two variables so that the feasible region can be graphed.

$14x + 4y \leq 40$ *(roses)*,
$9x + 7y \leq 80$ *(lilies)*,
$12x + 2y \leq 30$ *(carnations)*,
$y \leq 2x, \quad x, y \geq 0.$

$$P = 20x + 15y + 24x = 44x + 15y.$$



Because arrangements must be whole numbers, check the feasible integer points.

(x, y)	$(0,0)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$	$(2,3)$
P	0	44	59	74	88	103	118	133

The largest value occurs at $(2, 3)$*:*

$$P = 44(2) + 15(3) = 133.$$

Since $z = x$*,* $z = 2$*.*

Make 2 *arrangements X,* 3 *arrangements Y and* 2 *arrangements Z.* **(6)**

(b)(ii) Use the profit just found.

Maximum money: $\$133$*.* **(1)**

(c) With $z = x$*, the lilies constraint becomes* $9x + 7y \leq k$*. The previous optimum* $(x, y) = (2, 3)$ *remains feasible precisely when it satisfies this new constraint.*

$$9(2) + 7(3) \leq k \quad \Rightarrow \quad 39 \leq k.$$

$k \geq 39$. **(2)**