Maths with Matt

Physics and maths resources

Edexcel OCR MEI A Level Further Mathematics

H645/Y431 - Mechanics Minor

Wednesday 14 June 2023

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

(a) State the dimensions of force, velocity and density.

(b) The proposed drag formula is $F = ku^2(\rho m^2)^{1/3}$, where k is dimensionless. Show that it is dimensionally consistent.

(c) Explain why this formula conflicts with the observation that doubling ρ doubles F.

(d) For $F = k\rho^\alpha u^\beta A^\gamma$, where A is cross-sectional area, determine α, β, γ.

Model solution

(a) Write each quantity in the base dimensions M, L, T. Force is mass times acceleration, density is mass per volume, and velocity is distance per time.

$$[F] = MLT^{-2}, \qquad [u] = LT^{-1}, \qquad [\rho] = ML^{-3}.$$

$$[F] = MLT^{-2}, [u] = LT^{-1}, [\rho] = ML^{-3}.$$

(3)

(b) The test is to show that the dimensions of the right-hand side reduce to those of force; k contributes no dimensions.

$$[ku^2(\rho m^2)^{1/3}] = (LT^{-1})^2\left((ML^{-3})M^2\right)^{1/3} = L^2T^{-2}(M^3L^{-3})^{1/3} = MLT^{-2} = [F].$$

The formula is dimensionally consistent.

(2)

(c) Keep u and m fixed and look at the power of ρ. This directly predicts how F changes when density changes.

$$F \propto \rho^{1/3}.$$

Thus replacing ρ by 2ρ multiplies F by $2^{1/3}$, not by 2.

$$\text{The model predicts a factor } \sqrt[3]{2}, \text{ so it is inconsistent with the experiment.}$$

(1)

(d) Equate powers of the base dimensions on both sides. Here $[A] = L^2$, so dimensional homogeneity gives

$$MLT^{-2} = (ML^{-3})^\alpha (LT^{-1})^\beta (L^2)^\gamma = M^\alpha L^{-3\alpha+\beta+2\gamma} T^{-\beta}.$$

Matching the powers of M, T, L, respectively,

$$\alpha = 1, \qquad -\beta = -2 \Rightarrow \beta = 2, \qquad -3\alpha + \beta + 2\gamma = 1.$$

Hence $-3 + 2 + 2\gamma = 1$, so $\gamma = 1$.

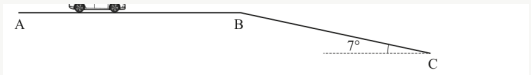
$\alpha = 1,\ \beta = 2,\ \gamma = 1, \text{hence} F = k\rho u^2 A.$

(3)

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Question 2

Question 2



Between A and B, the car moves at a constant speed of 12 m s^{-1}, and the power develope
car is a constant P W. When the car reaches B, the engine is switched off and the car trav

A 1400 kg car moves at 12 m s^{-1} on horizontal AB, with constant power P and constant resistance throughout. It then descends the 7° slope BC, engine off, accelerating at 0.8 m s^{-2}. (a) Find P. It returns up the slope at constant speed v, with power 16 kW. (b) Find v.

Model solution

(a) First use the downhill motion to find the unchanged resistance. Take downhill as positive. Newton's second law is appropriate because the acceleration is given.

$$1400g\sin 7^\circ - R = 1400(0.8), \qquad R = 1400g\sin 7^\circ - 1120 = 554.547 \text{ N}.$$

On horizontal AB, *constant speed means zero resultant force, so the driving force is* R. *Since power equals driving force times speed,*

$$P = R(12) = 554.547(12) = 6624.568 \text{ W}.$$

$$P = 6.62 \times 10^3 \text{ W} (\text{or} 6.62 \text{ kW}).$$

(4)

(b) Going uphill at constant speed again gives zero resultant force. The engine must overcome both the resistance and the component of weight down the slope.

$$\frac{16000}{v} - R - 1400g\sin 7^\circ = 0,$$

but the driving force is $16000/v$. *Therefore*

$$\frac{16000}{v} = 554.547 + 1400g\sin 7^\circ = 2224.568, \qquad v = 7.1939\ldots$$

$$v = 7.19 \text{ m s}^{-1}.$$

(2)

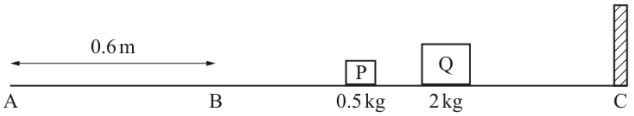
Question 3

Question 3

motion is unchanged.

(b) Determine the value of v.

The diagram shows two blocks P and Q of masses 0.5 kg and 2 kg respectively, on a horiz surface. The points A, B and C lie on the surface in a straight line. There is a wall at C. Tl surface between B and C is smooth, and the surface between A and B is rough, such that coefficient of friction between P and AB is $\frac{2}{3}$.



P is projected with a speed of 6 m s^{-1} directly towards Q, which is at rest. As a result of tl collision between P and Q, P changes direction and subsequently comes to rest at A. You assume that P only collides with Q once.

(a) Determine the coefficient of restitution between P and Q.

(b) Calculate the impulse exerted on P by Q during their collision.

After colliding with P, Q strikes the wall, which is perpendicular to the direction of the m Q, and comes to rest exactly halfway between A and B. The collision between Q and the perfectly elastic.

(c) Determine the coefficient of friction between Q and AB

Blocks P (0.5 kg) and Q (2 kg) lie on a horizontal line, with PQ $= 0.6$ m. The surface AB is rough and BC smooth; for P on AB, $\mu = \frac{2}{3}$. P is projected at 6 m s^{-1} towards stationary Q, then rebounds and stops at A. (a) Find the coefficient of restitution. (b) Find the impulse on P from Q. Q hits the wall elastically and stops halfway between A and B. (c) Find the coefficient of friction for Q on AB.

Model solution

(a) Before the collision, find P's speed at B using work against friction. The stated stopping point tells us that, after rebounding, its speed has exactly been removed by friction over 0.6 m.

$$F_f = \frac{2}{3}(0.5g).$$

If P's speed immediately after collision, towards A, is u, then

$$\frac{1}{2}(0.5)u^2 - \frac{2}{3}(0.5g)(0.6) = 0, \qquad u = 2.8 \text{ m s}^{-1}.$$

Take motion towards Q as positive and let Q's speed after collision be v. Conservation of linear momentum during the short collision gives

$$0.5(6) = 0.5(-2.8) + 2v, \qquad v = 2.2 \text{ m s}^{-1}.$$

The coefficient of restitution is separation speed divided by approach speed:

$$e = \frac{2.2 - (-2.8)}{6 - 0} = \frac{5}{6}.$$

$$e = \frac{5}{6} (= 0.833\ldots).$$

(6)

(b) Impulse is change in momentum. Retain the same positive direction, so P's final velocity is -2.8 m s^{-1}.

$$I = 0.5(-2.8 - 6) = -4.4 \text{ N s}.$$

The negative sign means it acts towards A.

Impulse magnitude 4.4 **N s**, towards A.

(2)

(c) The smooth section and a perfectly elastic wall collision do not change the speed, so Q returns to B at 2.2 m s^{-1}. It then loses all its kinetic energy to friction over 0.3 m.

$$\frac{1}{2}(2)(2.2)^2 = \mu(2g)(0.3).$$

$$\mu = \frac{4.84}{5.88} = \frac{121}{147} = 0.823129\ldots$$

$$\mu = \frac{121}{147} \approx 0.823.$$

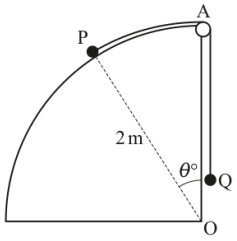
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Question 4

Question 4

to $\theta°$. The pulley, P and Q all lie in the same vertical plane.



Throughout this question you may assume that there are no re
Q and the force acting on P due to the tension in the string is t

(a) Given that P is in equilibrium at the point where $\theta = \alpha$, d

Particle P is now released from rest at the point on the surface
downwards on the surface. In the subsequent motion it is give

Masses P = 10 kg and Q = 5 kg are joined by a light inextensible string over a smooth pulley at the top of a smooth quadrant of radius 2 m. The angle AOP is θ. (a) If P is in equilibrium at θ = α, find α. From rest at 35°, P moves down and remains in contact. (b) Find its speed at 45°. (c) State one modelling assumption used in (b).

Model solution

(a) In equilibrium the common tension can be found from the hanging particle, then resolved tangentially for P.

$$T = 5g, \qquad T = 10g\sin\alpha.$$

$$5g = 10g\sin\alpha \quad \Rightarrow \quad \sin\alpha = \frac{1}{2}.$$

$$\alpha = 30°.$$

(3)

(b) Use conservation of mechanical energy because the surface and pulley are smooth. Between $35°$ *and* $45°$, P *falls by* $2(\cos 35° - \cos 45°)$*; the inextensible string makes* Q *rise by the arc length* $2 \times \frac{10\pi}{180} = \frac{\pi}{9}$.

$$10g\{2\cos 35° - 2\cos 45°\} = \frac{1}{2}(10)v^2 + \frac{1}{2}(5)v^2 + 5g\left(\frac{\pi}{9}\right).$$

$$v^2 = 0.64755\ldots, \qquad v = 0.804706\ldots$$

$$v = 0.805 \ m\ s^{-1}.$$

(4)

(c) The energy calculation treats the string displacement as the arc length travelled by P*, which requires the particle and pulley to be negligible in size.*

Forexample,thestringsegmentAPistakentohavethesamechangeinlengthasth

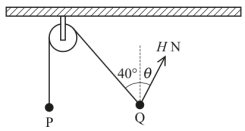
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Question 5

Question 5

Fig. 5.1

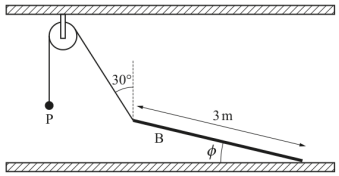


When a force of magnitude H N, acting at an angle θ to the upward vertical, is applied to Q the particles hang in equilibrium, with the part of the string connecting the pulley to Q making an angle of 40° with the upward vertical. It is given that the force acts in the same vertical plane in which the string lies.

(a) Determine the values of H and θ. **[4]**

Particle Q is now removed. The string is instead attached to one end of a uniform beam B of length 3 m and mass 7 kg. The other end of B is in contact with a rough horizontal floor. The situation is shown in **Fig. 5.2**.

Fig. 5.2



With B in equilibrium, at an angle ϕ to the horizontal, the part of the string connecting the pulley to B makes an angle of 30° with the upward vertical.

It is given that the string and B lie in the same vertical plane.

(b) Determine the smallest possible value for the coefficient of friction between B and the floor. **[3]**

(c) Determine the value of ϕ. **[5]**

Particles P $= 5$ kg and Q $= 11$ kg hang from a light string over a smooth pulley. A force H, at angle θ to the upward vertical, holds them in equilibrium while the string to Q is at 40° to the upward vertical. (a) Find H, θ. Then Q is replaced by a uniform 3 m, 7 kg beam, with its other end on a rough horizontal floor. The string to the beam is at 30° to the upward vertical. (b) Find the least coefficient of friction. (c) Find the angle ϕ of the beam to the horizontal.

Model solution

(a) The tension is $5g$, from the equilibrium of P. Resolve the forces on Q horizontally and vertically; this is preferable to taking moments because all three forces meet at Q.

$$H\sin\theta = 5g\sin 40^\circ, \qquad H\cos\theta + 5g\cos 40^\circ = 11g.$$

Dividing the first equation by the second after rearranging gives

$$\tan\theta = \frac{5\sin 40^\circ}{11 - 5\cos 40^\circ}, \qquad \theta = 24.14487\ldots^\circ.$$

Then

$$H = \frac{5g\sin 40^\circ}{\sin\theta} = 77.000\ldots \text{ N}.$$

$$H = 77.0 \text{ N}, \theta = 24.1^\circ.$$

(4)

(b) Consider the beam's force balance. Its tension remains $5g$; its horizontal component is balanced by friction and its upward vertical component reduces the normal reaction. At limiting equilibrium, $F = \mu R$.

$$F = 5g\sin 30^\circ, \qquad R = 7g - 5g\cos 30^\circ.$$

$$\mu_{\min} = \frac{F}{R} = \frac{5\sin 30^\circ}{7 - 5\cos 30^\circ} = 0.936374\ldots$$

$$\mu_{\min} = 0.936 (3\text{s.f.}).$$

(3)

(c) Take moments about the floor contact. This eliminates both unknown contact forces. The vertical component of tension has moment arm $3\cos\phi$; its horizontal component and the beam's weight give clockwise moments.

$$5g\cos 30^\circ (3\cos\phi) = 5g\sin 30^\circ (3\sin\phi) + 7g\left(\frac{3}{2}\cos\phi\right).$$

After cancelling $3g$ and rearranging,

$$5\cos(\phi + 30^\circ) = \frac{7}{2}\cos\phi, \qquad \tan\phi = \frac{5\sqrt{3} - 7}{5}.$$

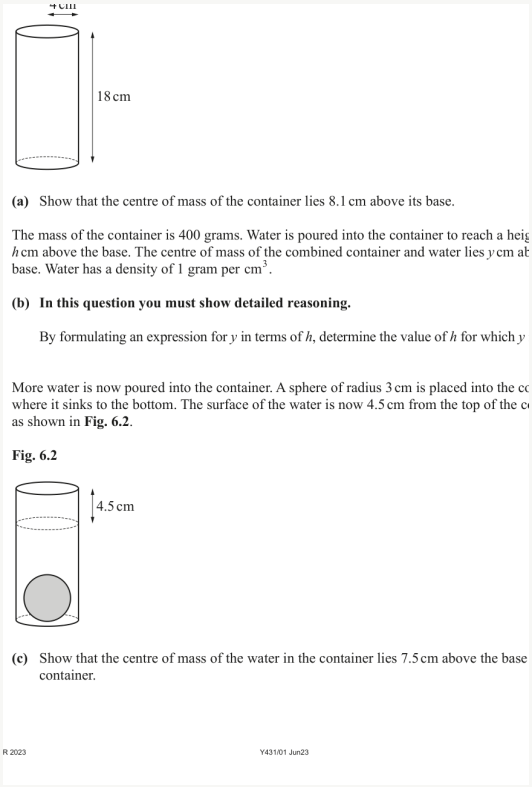
$$\phi = 18.36878\ldots^\circ.$$

$$\phi = 18.4^\circ (3\text{s.f.}).$$

(5)

Question 6

Question 6



An open cylindrical container has radius 4 cm, height 18 cm, uniform surface density, and mass 400 g. (a) Show its centre of mass is 8.1 cm above the base. Water of density 1 g cm^{-3} is poured to height h. (b) Find the h that minimises the height y of the combined centre of mass. A radius-3 cm, density-4 g cm^{-3} sphere then sinks to the bottom; water is 4.5 cm below the top. (c) Show the water centre is at 7.5 cm. (d) Find the combined centre z.

Model solution

(a) Use moments of area because the laminae have the same mass per unit area. The base has area 16π at height 0, and the curved surface has area $2\pi(4)(18) = 144\pi$ with centre at height 9.

$$x(160\pi) = 16\pi(0) + 144\pi(9), \qquad x = 8.1.$$

The container's centre of mass is 8.1 cm above its base.

(3)

(b) The water has volume, hence mass, $16\pi h$, and its centre is at height $h/2$. Form the moment equation before differentiating, since y is a weighted mean.

$$(400 + 16\pi h)y = 400(8.1) + (16\pi h)\left(\frac{h}{2}\right),$$

$$y = \frac{3240 + 8\pi h^2}{400 + 16\pi h} = \frac{405 + \pi h^2}{50 + 2\pi h}.$$

To minimise this quotient,

$$\frac{dy}{dh} = \frac{(2\pi h)(50 + 2\pi h) - 2\pi(405 + \pi h^2)}{(50 + 2\pi h)^2}.$$

The denominator is positive, so set the numerator to zero:

$$2\pi h(50 + 2\pi h) - 2\pi(405 + \pi h^2) = 0 \;\Rightarrow\; \pi h^2 + 50h - 405 = 0.$$

The physically valid root is

$$h = 5.90736\ldots$$

and the derivative changes from negative to positive there, so this is the minimum.

$$h = 5.91 \text{ cm} (3\text{s.f.}).$$

(6)

(c) The water reaches height 13.5 *cm. Treat it as a full cylinder of water minus the spherical volume, whose centre is* 3 *cm above the base. This accounts for the displaced water.*

$$\big(16\pi(13.5) - 36\pi\big)\,d = 16\pi(13.5)\left(\frac{13.5}{2}\right) - 36\pi(3).$$

$$180\pi d = 1350\pi \quad\Rightarrow\quad d = 7.5.$$

$$\text{The water's centre of mass is } 7.5 \text{ cm above the base.}$$

(2)

(d) Combine the moments of the container, water and sphere. The sphere has volume 36π *cm*^{3}*, so its mass is* $4(36\pi) = 144\pi$ *g; the water mass is* 180π *g.*

$$(400 + 180\pi + 144\pi)z = 400(8.1) + 180\pi(7.5) + 144\pi(3).$$

$$z = \frac{3240 + 1782\pi}{400 + 324\pi} = 6.233491\ldots$$

$$z = 6.23 \text{ cm} (3\text{s.f.}) \text{ above the base.}$$

(3)