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# Edexcel OCR MEI A Level Further Mathematics

## H645/04 - Mechanics Minor

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## *Questions and Fully Worked Model Solutions*

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2024 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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# Question 1

## Question 1

A car of mass 1500 kg moves on a horizontal straight road with no resistances. Its constant power is 5000 W while its speed increases from 20 to 30 $\mathrm{m\,s^{-1}}$ in t seconds.
(a) Find t. **(3)**
(b) Find its acceleration when its speed is 25 $\mathrm{m\,s^{-1}}$. **(2)**

## *Model solution*

*(a) With no resistance and no change in height, all of the work done becomes kinetic energy. Constant power is work done per second.*

$$\Delta \mathrm{KE} = \frac{1}{2}(1500)\left(30^2 - 20^2\right) = 375000 \text{ J}.$$

$$5000 = \frac{375000}{t} \quad \Rightarrow \quad t = 75.$$

$$t = 75 \text{ s}.$$

*(b) At one particular speed, use* $P = Fv$ *to find the instantaneous driving force, then apply* $F = ma$.

$$F = \frac{P}{v} = \frac{5000}{25} = 200 \text{ N}, \qquad a = \frac{F}{m} = \frac{200}{1500} = \frac{2}{15}.$$

$$a = \frac{2}{15} \ \mathrm{m\,s^{-2}} \ \textit{(about } 0.133 \ \mathrm{m\,s^{-2}}\textit{)}.$$

# Question 2

## Question 2

(a) State the dimensions of force. **(1)**
For the remaining parts, use $1\,\text{kg} = 2.2\,\text{lb}$ and $1\,\text{m} = 39.4\,\text{in}$. One pound-force is the gravitational force on 1 lb at Earth's surface.
(b) Show that $1\,\text{lbf} \approx 4.45\,\text{N}$. **(1)**
(c) A diver at depth 40 m experiences pressure $5 \times 10^5\,\text{Pa}$. Decide whether this is greater or less than 80 psi. **(3)**
(d) Show that energy density and pressure are dimensionally equivalent. **(3)**

## *Model solution*

*(a) Force is mass times acceleration, so its dimensions follow directly from* $\mathrm{F} = \mathrm{ma}$.

$$[\mathrm{F}] = \mathrm{M}(\mathrm{LT}^{-2}) = \mathrm{MLT}^{-2}.$$

$$[\mathrm{F}] = \mathrm{MLT}^{-2}.$$

*(b) Convert the mass of one pound to kilograms, then use weight* $\mathrm{mg}$.

$$1\,\text{lb} = \frac{1}{2.2}\,\text{kg}, \qquad 1\,\text{lbf} = \frac{9.8}{2.2}\,\text{N} = 4.4545\ldots\,\text{N}.$$

$$1\,\text{lbf} \approx 4.45\,\text{N}.$$

*(c) Convert the diver's pressure from newtons per square metre into pound-force per square inch, converting both the force and the area.*

$$5 \times 10^5\,\text{Pa} = \frac{5 \times 10^5}{4.45 \times 39.4^2}\,\text{psi} = 72.3\ldots\,\text{psi}.$$

$72.3\,\text{psi} < 80\,\text{psi}$, *so the diver's pressure is less than the bicycle-tyre pressure.*

*(d) Pressure is force divided by area. Energy density is energy divided by volume; obtain energy dimensions from work = force × distance.*

$$[\mathrm{p}] = \frac{\mathrm{MLT}^{-2}}{\mathrm{L}^2} = \mathrm{ML}^{-1}\mathrm{T}^{-2}, \qquad [\mathrm{E}] = [\mathrm{Fd}] = (\mathrm{MLT}^{-2})\mathrm{L} = \mathrm{ML}^2\mathrm{T}^{-2},$$

$$[\textit{energy density}] = \frac{\mathrm{ML}^2\mathrm{T}^{-2}}{\mathrm{L}^3} = \mathrm{ML}^{-1}\mathrm{T}^{-2} = [\mathrm{p}].$$

*Energy density and pressure are dimensionally equivalent.*

# Question 3

## Question 3

A 6 kg block is projected from B up a rough plane inclined at 25°. It travels 3.5 m to C, where it stops, then slides down. Its initial speed is $u$, and on the way down it regains this speed at A, which is $x$ m below B. Resistance does 4 J of work per metre throughout.
(a)(i) Using energy, find $u$. **(3)**
(a)(ii) Using energy, find $x$. **(3)**
(b) If half the resistance is friction with the plane and half is air resistance, find the coefficient of friction. **(3)**

## *Model solution*

*(a)(i) The block stops at C, so its initial kinetic energy is used in gaining gravitational potential energy and in working against resistance. This is an energy calculation because the distance travelled is given.*

$$\frac{1}{2}(6)u^2 = (6)(9.8)(3.5\sin 25°) + 4(3.5).$$

$$u^2 = 33.658\ldots \quad \Rightarrow \quad u = 5.80\ldots$$

$$u = 5.80 \text{ m s}^{-1}.$$

*(a)(ii) From C to A, the block falls a distance $(3.5 + x)\sin 25°$. That lost potential energy supplies the regained kinetic energy and the resistance over distance $3.5 + x$.*

$$6g(3.5 + x)\sin 25° = \frac{1}{2}(6)u^2 + 4(3.5 + x).$$

$$6(9.8)(3.5 + x)\sin 25° = 3(5.801575\ldots)^2 + 4(3.5 + x),$$

$$x = 1.3429\ldots$$

$$x = 1.34 \text{ m}.$$

*(b) Half of the $4\,\text{J m}^{-1}$ resistance is friction. Since work per metre equals force, the frictional force is 2 N. The normal reaction is $6g\cos 25°$.*

$$\mu = \frac{\textit{friction}}{\textit{normal reaction}} = \frac{2}{6(9.8)\cos 25°} = 0.03753\ldots$$

$$\mu = 0.0375 \textit{ (approximately } 0.038\textit{)}.$$

# Question 4

## Question 4

On a smooth horizontal surface, sphere A (mass 3 kg) moves at $1\,\text{m s}^{-1}$ towards stationary sphere B (mass 1 kg). Their coefficient of restitution is $e$.

(a) Show that the speed of A after impact is $\frac{3-e}{4}\,\text{m s}^{-1}$, and find the speed of B. **(4)**

(b) The kinetic energy decreases by 21%. Find $e$. **(3)**

(c) Explain why equal radii were needed in (a). **(1)**

Spheres C and D, of equal radii and masses 1 kg and 2 kg, move towards each other with speeds $u$ and $1\,\text{m s}^{-1}$ respectively. The impulse exerted by C on D has magnitude $\frac{2}{3}(u+1)\,\text{N s}$.

(d) Show that they have the same velocity after impact. **(4)**

(e) Find the limiting fraction of kinetic energy lost as $u \to \infty$. **(3)**

## *Model solution*

*(a) Take the positive direction from A to B. The collision is direct, so conservation of linear momentum and restitution give two equations for the two final velocities $v_A, v_B$.*

$$3(1) = 3v_A + v_B, \qquad v_B - v_A = e(1-0) = e.$$

$$3v_A + (v_A + e) = 3 \quad \Rightarrow \quad v_A = \frac{3-e}{4},$$

$$v_B = v_A + e = \frac{3(1+e)}{4}.$$

A *has speed* $\frac{3-e}{4}\,\text{m s}^{-1}$ *and* B *has speed* $\frac{3(1+e)}{4}\,\text{m s}^{-1}$.

*(b) A* 21% *loss means* 79% *of the initial kinetic energy remains. Substitute the velocities just found.*

$$\frac{1}{2}(3)\left(\frac{3-e}{4}\right)^2 + \frac{1}{2}\left(\frac{3(1+e)}{4}\right)^2 = 0.79\left[\frac{1}{2}(3)(1)^2\right].$$

$$\frac{3\left[(3-e)^2 + 3(1+e)^2\right]}{32} = 1.185 \quad \Rightarrow \quad e^2 = 0.16.$$

$e = 0.4$ *because a coefficient of restitution is non-negative.*

$$e = 0.4.$$

*(c) Equal radii ensure that the line of centres at impact is the line of motion, so the impact is direct and there is no vertical impulse.*

*With unequal radii the impact need not be direct, so the one-dimensional collision equations used in (a) would not apply.*

*(d) Take positive direction from C towards D. Sphere D initially has velocity $-1$. Apply the stated impulse to D, then use conservation of momentum.*

$$\frac{2}{3}(u+1) = 2\big(v_D - (-1)\big) \quad \Rightarrow \quad v_D = \frac{u-2}{3}.$$

$$u - 2 = v_C + 2v_D \quad \Rightarrow \quad v_C = u - 2 - 2\left(\frac{u-2}{3}\right) = \frac{u-2}{3}.$$

$v_C = v_D = \dfrac{u-2}{3}$, *so the spheres have the same velocity after impact.*

*(e) Since the spheres move together afterwards, use their common velocity to form final energy divided by initial energy, then take the limit.*

$$\frac{KE_{final}}{KE_{initial}} = \frac{\frac{1}{2}(3)\left(\frac{u-2}{3}\right)^2}{\frac{1}{2}u^2 + \frac{1}{2}(2)(1)^2} = \frac{u^2 - 4u + 4}{3u^2 + 6}.$$

$$\lim_{u\to\infty} \frac{u^2 - 4u + 4}{3u^2 + 6} = \frac{1}{3}.$$

*The fraction lost tends to* $1 - \frac{1}{3} = \dfrac{2}{3}$.

# Question 5

## Question 5

Uniform lamina OABC is a trapezium with $O = (0,0)$, $A = (120,0)$, $B = (60,90)$ and $C = (30,90)$, in centimetres. Its centre of mass is $(\bar{x}, \bar{y})$.

(a) Show that $\bar{x} = 54$ and find $\bar{y}$. **(5)**

The horizontal lamina rests on supports at B, C and $D = (d,0)$ on OA.

(b) Find the range of d for equilibrium. **(3)**

(c) Given $d = 63$ and weight 100 N, find the support forces at B, C, D. **(4)**

## *Model solution*

*(a) Split the trapezium into a left triangle, a central rectangle and a right triangle. Their areas are* $1350$, $2700$ *and* $2700\,\text{cm}^2$ *and their centroids are* $(20,30)$, $(45,45)$ *and* $(80,30)$ *respectively. The centre of mass is the area-weighted average of these centroids.*

$$A = \frac{1}{2}(30+120)(90) = 6750\ \text{cm}^2,$$

$$(\bar{x}, \bar{y}) = \frac{1}{6750}\left[1350(20,30) + 2700(45,45) + 2700(80,30)\right].$$

$$\bar{x} = 54, \qquad \bar{y} = 36.$$

*The centre of mass is* $(54,36)\,\text{cm}$.

*(b) For vertical support reactions to be possible, the vertical line through the centre of mass must lie inside the support triangle* BCD. *At the limiting positions,* $(54,36)$ *lies on* BD *or* CD.

$$\textit{Line BD}: \quad \frac{36-0}{90-0} = \frac{54-d}{60-d} \quad \Rightarrow \quad d = 50,$$

$$\textit{Line CD}: \quad \frac{36-0}{90-0} = \frac{54-d}{30-d} \quad \Rightarrow \quad d = 70.$$

$$50 \leqslant d \leqslant 70\ \text{cm}.$$

*(c) Let the upward reactions be* $R_B, R_C, R_D$. *First use vertical equilibrium. Then moments about horizontal and vertical axes through convenient support lines isolate* $R_D$ *and* $R_B$.

$$R_B + R_C + R_D = 100.$$

$$\textit{Moments about } y = 90: \qquad 90R_D = 54(100), \qquad R_D = 60.$$

$$\textit{Moments about } x = 30: \qquad 30R_B + 33R_D = 24(100).$$

$$30R_B + 33(60) = 2400 \quad \Rightarrow \quad R_B = 14, \qquad R_C = 100 - 14 - 60 = 26.$$

$R_B = 14\,N,\ R_C = 26\,N,\ R_D = 60\,N$, ***all upwards.***

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# Question 6

## Question 6

(a) A horizontal 6 m beam AB has downward 15 N forces at A and B, and an upward 30 N force x m from B. The forces form a clockwise couple of magnitude 42 N m. Find x. **(2)**

The beam is then placed at $60°$ to a rough horizontal floor at B, with its other contact C against a rough step. C is 1.5 m from A, and the centre of mass is 3 m from B.

(b) Draw all forces on the beam. **(2)**

(c)(i) The coefficient of friction at both contacts is $\mu$. Show that $\mu^2 - 6\mu + 1 = 0$. **(5)**

(c)(ii) Hence find $\mu$. **(2)**

## *Model solution*

*(a) Take moments about* B*: the force at* B *has no moment. Clockwise is taken as positive, as specified by the couple.*

$$30x - 15(6) = 42 \quad \Rightarrow \quad x = 4.4.$$

$$x = 4.4\,\text{m}.$$

*(b) The limiting-equilibrium clue means friction is at its maximum at both contacts. A complete force diagram has: weight* $W$ *vertically downward at the centre of mass; normal reaction* $R_B$ *vertically upward at* B *and friction* $F_B$ *horizontally towards the step; normal reaction* $R_C$ *at* C*, perpendicular to the beam and away from the step; and friction* $F_C$ *at* C*, parallel to the beam towards* A*.*

*These five labelled forces form the required free-body diagram.*

*(c)(i) Resolve horizontally and vertically, taking the force directions from the free-body diagram. Taking moments about* B *removes both forces at* B*.*

$$R_C(4.5) = W(3\cos 60°) \quad \Rightarrow \quad R_C = \frac{W}{9}.$$

$$R_B + R_C \sin 30° + F_C \sin 60° = W, \qquad R_C \cos 30° = F_C \cos 60° + F_B.$$

*At limiting equilibrium,* $F_B = \mu R_B$ *and* $F_C = \mu R_C$*. Eliminate* $R_B$ *using the vertical equation.*

$$\frac{\sqrt{3}}{2}R_C - \frac{1}{2}F_C = \mu\left(W - \frac{1}{2}R_C - \frac{\sqrt{3}}{2}F_C\right),$$

*Substitute* $F_C = \mu R_C$*,* $F_B = \mu R_B$ *and the value of* $R_C$ *into these equations, then simplify.*

$$3 = 18\mu - 3\mu^2 \quad \Rightarrow \quad \mu^2 - 6\mu + 1 = 0.$$

$$\mu^2 - 6\mu + 1 = 0, \textit{ as required.}$$

*(c)(ii) Solve the quadratic, then reject the root that would make a contact reaction negative.*

$$\mu = \frac{6 \pm \sqrt{36-4}}{2} = 3 \pm 2\sqrt{2}.$$

$3 + 2\sqrt{2}$ *gives* $R_B < 0$, *which is physically impossible for an upward floor reaction.*

$$\mu = 3 - 2\sqrt{2} = 0.1715728\ldots.$$

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