Maths with Matt

Physics and maths resources

Edexcel OCR MEI A Level Further Mathematics

H645/Y431 - Mechanics Minor

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Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Particles A and B collide directly on a smooth horizontal surface. Before impact their speeds towards one another are 3 and 8 m s^{-1}; afterwards they move apart, A at 5.2 m s^{-1} and B at v m s^{-1}. The collision is perfectly elastic.

(a) Find v. **(1)**

(b) State the kinetic energy lost. **(1)**

Particles C and D then collide directly. C approaches D at 6.4 m s^{-1}, and D is moving away from C at 1.9 m s^{-1}. Afterwards C moves away at 0.2 m s^{-1}. Given $e = 0.8$ and that C has mass 3.5 kg, determine the mass of D. **(4)**

Model solution

(a) For a perfectly elastic direct collision, the relative speed of separation equals the relative speed of approach. This is the quickest way to use the word "perfectly elastic".

$$5.2 + v = 3 + 8 = 11, \qquad v = 5.8.$$

$$v = 5.8 \text{ m s}^{-1}.$$

(1)

(b) A perfectly elastic collision has no loss of kinetic energy.

$$\text{Kinetic energy lost} = 0 \text{ J}.$$

(1)

(c) Take the initial direction of C as positive. The coefficient of restitution relates the relative speeds along the line of impact; after impact D moves faster than C, so its velocity is positive and C's is -0.2.

$$e = \frac{\textit{speed of separation}}{\textit{speed of approach}} = \frac{v_D - (-0.2)}{6.4 - 1.9}.$$

$$0.8 = \frac{v_D + 0.2}{4.5} \quad \Rightarrow \quad v_D = 3.4 \text{ m s}^{-1}.$$

The surface is smooth, so there is no external horizontal impulse and momentum is conserved. Let the mass of D be m kg.

$$3.5(6.4) + m(1.9) = 3.5(-0.2) + m(3.4).$$

$$22.4 + 1.9m = -0.7 + 3.4m \quad \Rightarrow \quad 1.5m = 23.1 \quad \Rightarrow \quad m = 15.4.$$

$$\text{The mass of D is } 15.4 \text{ kg}.$$

(4)

Question 2

Question 2

(a)(i) State the dimensions of force. **(1)**
(a)(ii) State the dimensions of energy. **(1)**
Surface tension S is force per unit length and also energy per unit area.
(b) Show that these definitions are dimensionally consistent. **(2)**
The maximum height h of a droplet is modelled by $h = mS^a g^b \rho^c$, where m is dimensionless, g is acceleration due to gravity and ρ is density. Determine a, b, c. **(4)**
For water, $\rho = 1\ \text{g cm}^{-3}$, $S = 0.073\ \text{N m}^{-1}$, and $h = 0.53$ cm. Find m. **(2)**

Model solution

(a)(i) Start from $F = ma$.

$$[F] = MLT^{-2}.$$

(1)

(a)(ii) Energy is force multiplied by distance.

$$[E] = ML^2T^{-2}.$$

(1)

(b) Substitute these dimensions into each definition, reducing both to powers of M, L, T.

$$\left[\frac{\textit{force}}{\textit{length}}\right] = \frac{MLT^{-2}}{L} = MT^{-2}, \qquad \left[\frac{\textit{energy}}{\textit{area}}\right] = \frac{ML^2T^{-2}}{L^2} = MT^{-2}.$$

$$\text{Both definitions give } [S] = MT^{-2}, \text{ so they are dimensionally consistent.}$$

(2)

(c) Dimensional analysis is appropriate because m *has no dimensions. The dimensions on the right must therefore be exactly those of height.*

$$[h] = L, \qquad [S] = MT^{-2}, \qquad [g] = LT^{-2}, \qquad [\rho] = ML^{-3}.$$

$$L = (MT^{-2})^a (LT^{-2})^b (ML^{-3})^c = M^{a+c} L^{b-3c} T^{-2a-2b}.$$

Equating the exponents of mass, length and time gives three equations for the three unknown powers.

$$a + c = 0, \qquad b - 3c = 1, \qquad -2a - 2b = 0.$$

$$a = \frac{1}{2}, \qquad b = -\frac{1}{2}, \qquad c = -\frac{1}{2}.$$

$$a = \tfrac{1}{2},\ b = -\tfrac{1}{2},\ c = -\tfrac{1}{2}.$$

(4)

(d) Use SI units before substituting: $h = 0.0053\ \text{m}$ ***and*** $\rho = 1000\ \text{kg}\,\text{m}^{-3}$. ***The model now reads*** $h = m\sqrt{S/(g\rho)}$.

$$m = \frac{0.0053}{\sqrt{\dfrac{0.073}{9.8 \times 1000}}} = 1.94 \quad \textit{(to 3 s.f.)}.$$

$$m \approx 1.94.$$

(2)

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Question 3

Question 3

A 30 kg crate is pulled up a smooth inclined plane by a light rope over a smooth pulley. It rises vertically 3.5 m from A to B and is at rest at both points.

(a) Calculate the work done by the tension. **(1)**

At C the rope is at 12° to the direction of motion, its tension is 206 N, and the crate speed is 0.8 m s^{-1}.

(b) Calculate the power of the tension. **(2)**

The crate subsequently slides from rest at B to A; 894 J of work is done against the tension.

(c) Calculate its speed at A. **(2)**

(d) State one limitation of the model which could affect part (c). **(1)**

Model solution

(a) The crate starts and finishes at rest, so its gain in gravitational potential energy is supplied by the work done by the tension. The plane is smooth, so there is no frictional work.

$$W_T = mg\Delta h = 30 \times 9.8 \times 3.5 = 1029 \text{ J}.$$

1029 J.

(1)

(b) Power is the component of force in the direction of velocity multiplied by speed. The stated 12° is the angle between the tension and the motion.

$$P = Fv\cos\theta = 206(0.8)\cos 12^\circ = 161.2\ldots \text{ W}.$$

161 W(3s.f.).

(2)

(c) Apply work--energy from B to A. The loss of gravitational potential energy is 1029 J; 894 J is removed by tension, leaving kinetic energy.

$$1029 - 894 = \frac{1}{2}(30)v^2, \qquad 135 = 15v^2, \qquad v = 3.$$

Speed $= 3.0$ m s^{-1}.

(2)

(d) The model calls the plane smooth, so it neglects friction; in practice friction (or air resistance) would change the energy available.

For example, friction/air resistance has been neglected.

(1)

Question 4

Question 4

A block of mass m rests on a rough plane at angle α, with coefficient of friction μ. As α is increased:
(a) show that, if it does not topple first, sliding begins when $\mu = \tan\alpha$. **(2)**
The block is a uniform rectangular prism with cross-section width w cm and height 22 cm, with the cross-section in a vertical plane of greatest slope. Given $\mu = 0.6$, and that it slides rather than topples, determine the range of w. **(3)**
For the rest of the question, $w = 15$ cm, $m = 10$ kg, and $\alpha = 30°$. A string attached to the upper edge makes $45°$ with the plane and holds the block with its base horizontal.
(c) Determine the tension. **(3)**
(d) Determine the angle between the plane and the total contact force. **(6)**

Model solution

(a) At impending sliding, friction has reached its limiting value $F = \mu R$. *Resolve weight perpendicular and parallel to the plane.*

$$R = mg\cos\alpha, \qquad F = mg\sin\alpha.$$

$$mg\sin\alpha = \mu mg\cos\alpha \quad \Rightarrow \quad \mu = \tan\alpha.$$

$$\mu = \tan\alpha \text{at the point of sliding.}$$

(2)

(b) Sliding would begin at $\alpha = \tan^{-1}(0.6)$. *At the tipping threshold, take moments about the downhill lower edge: the vertical line through the centre of mass is then through that edge. Since the width is along the plane and the height is normal to it, this gives* $\tan\alpha = w/22$.

$$\tan^{-1}(0.6) < \tan^{-1}\left(\frac{w}{22}\right).$$

For sliding to occur first, its critical angle must be the smaller one. Hence

$$0.6 < \frac{w}{22} \quad \Rightarrow \quad w > 13.2.$$

$$w > 13.2 \text{ cm}.$$

(3)

(c) Take moments about the lower edge of the block. This eliminates the unknown contact force. The weight acts 7.5 cm *horizontally from that edge, and the perpendicular distance for the tension is* $22\sin 75°$ cm.

$$T(22\sin 75°) = 10g(7.5).$$

$$T = \frac{10(9.8)(7.5)}{22\sin 75°} = 34.59\ldots$$

$$T = 34.6 \text{ N} (3\text{s.f.}).$$

(3)

(d) Let F be the frictional component of the contact force, acting up the plane, and R its normal component. Resolving is appropriate because these are perpendicular components of the required resultant.

$$F + T\cos 45^\circ = 10g \sin 30^\circ,$$

$$F = 10(9.8)\sin 30^\circ - 34.59\ldots \cos 45^\circ = 24.54\ldots \text{ N}.$$

$$R = 10g\cos 30^\circ + T\sin 45^\circ = 109.33\ldots \text{ N}.$$

The contact force has components F along the plane and R normal to it, so its angle θ to the plane satisfies $\tan\theta = R/F$.

$$\theta = \tan^{-1}\left(\frac{109.33\ldots}{24.54\ldots}\right) = 77.35\ldots^\circ.$$

$$\text{The angle is } 77.3^\circ (3\text{s.f.}).$$

(6)

Question 5

Question 5

A light cuboid frame has $O = (0,0,0)$, $A = (50,0,0)$, $C = (0,30,0)$, and $E = (0,0,20)$. Particles of masses 9 kg, 5 kg, and 6 kg are fixed at $D = (0,30,20)$, $G = (50,30,20)$, and $(\lambda,0,0)$ on OA respectively, where $0 \leqslant \lambda \leqslant 50$. The centre of mass is $(\bar{x},\bar{y},\bar{z})$.

(a) Show that $\bar{y} = 21$ and $\bar{z} = 14$. **(2)**

(b) Find the range of $\bar{x}$. **(3)**

The 6 kg particle is now at A. The frame is suspended by vertical wires at O and C, with OC horizontal.

(c) Find the two tensions. **(3)**

(d) Find the angle the plane OABC makes with the downward vertical. **(2)**

The wire at C snaps, leaving only the vertical wire at O.

(e) Find the angle OA makes with the downward vertical. **(2)**

Model solution

(a) The frame is light, so only the three particles contribute. Use mass-weighted coordinates and divide by total mass $9+5+6=20$.

$$\bar{y} = \frac{9(30)+5(30)+6(0)}{20} = 21, \qquad \bar{z} = \frac{9(20)+5(20)+6(0)}{20} = 14.$$

$$\bar{y} = 21, \quad \bar{z} = 14.$$

(2)

(b) Apply the same centre-of-mass formula to the x-coordinates. Only G and the 6 kg particle have non-zero x-coordinate.

$$\bar{x} = \frac{5(50)+6\lambda}{20} = \frac{250+6\lambda}{20}.$$

This expression increases with λ, so its extreme values occur at the endpoints 0 and 50.

$$\bar{x} = 12.5 \quad (\lambda = 0), \quad \bar{x} = 27.5 \quad (\lambda = 50).$$

$$12.5 \leqslant \bar{x} \leqslant 27.5.$$

(3)

(c) With the 6 kg mass at A, $\bar{x} = 27.5$. Let the upward tensions at O and C be T_O and T_C. Take moments about O; this removes T_O, and the perpendicular lever arms are the y-coordinates.

$$30T_C = (20g)(21) \quad \Rightarrow \quad T_C = 137.2 \text{ N}.$$

$$T_O + T_C = 20g = 196 \quad \Rightarrow \quad T_O = 58.8 \text{ N}.$$

$$T_O = 58.8 \text{ N}, T_C = 137.2 \text{ N}.$$

(3)

(d) The frame rotates about the horizontal line OC until the centre of mass is vertically below that line. In the cross-section perpendicular to OC, its offsets are 27.5 in the plane OABC and 14 normal to it.

$$\theta = \tan^{-1}\left(\frac{14}{27.5}\right) = 26.98\ldots^\circ.$$

$27.0^\circ (3\text{s.f.}).$

(2)

(e) With one wire, the centre of mass must lie directly below O. Therefore OA makes the same angle with the downward vertical as OA makes with the vector from O to the centre of mass. The component of that vector perpendicular to OA has size $\sqrt{21^2 + 14^2}$.

$$\theta = \tan^{-1}\left(\frac{\sqrt{21^2 + 14^2}}{27.5}\right) = 42.54\ldots^\circ.$$

$42.5^\circ (3\text{s.f.}).$

(2)

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Question 6

Question 6

The horizontal unit vectors are $\mathbf{i}, \mathbf{j}$. Amos and Beth have masses 70 kg and 50 kg. Initially they move together at $4\mathbf{i}\ \mathrm{m\,s^{-1}}$. Amos exerts an impulse $(35\mathbf{i} + 70\mathbf{j})\ \mathrm{N\,s}$ on Beth.
(a) Determine Beth's speed after separation. **(3)**
(b) Determine the total gain in kinetic energy. **(3)**
On another occasion they initially move together at $(3.5\mathbf{i} + 2.5\mathbf{j})\ \mathrm{m\,s^{-1}}$. After Amos exerts an impulse, they have equal speeds and Amos travels in the $\mathbf{i}$ direction. Determine the possible speeds. **(6)**

Model solution

(a) Use impulse equals change in momentum for Beth. The given impulse is the impulse on Beth, so add it to Beth's initial momentum.

$$50\mathbf{v}_B = 50(4\mathbf{i}) + (35\mathbf{i} + 70\mathbf{j}).$$

$$\mathbf{v}_B = 4.7\mathbf{i} + 1.4\mathbf{j}.$$

$$|\mathbf{v}_B| = \sqrt{4.7^2 + 1.4^2} = 4.90\ \mathrm{m\,s^{-1}}.$$

$$\mathrm{Beth's speed is } 4.90\ \mathrm{m\,s^{-1}}.$$

(3)

(b) Amos receives the equal and opposite impulse. This finds his velocity before calculating the kinetic-energy change of the whole two-skater system.

$$70\mathbf{v}_A = 70(4\mathbf{i}) - (35\mathbf{i} + 70\mathbf{j}), \qquad \mathbf{v}_A = 3.5\mathbf{i} - \mathbf{j}.$$

$$\Delta\mathrm{KE} = \frac{1}{2}(70)(3.5^2 + 1^2) + \frac{1}{2}(50)(4.7^2 + 1.4^2) - \frac{1}{2}(120)(4^2).$$

$$\Delta\mathrm{KE} = 105\ \mathrm{J}.$$

$$\mathrm{Total gain in kinetic energy} = 105\ \mathrm{J}.$$

(3)

(c) Let Amos's final velocity be $u\mathbf{i}$, since he travels in the $\mathbf{i}$ direction, and let Beth's be $v\mathbf{i} + w\mathbf{j}$. There is no external horizontal impulse, so conserve momentum in each horizontal direction.

$$120(3.5\mathbf{i} + 2.5\mathbf{j}) = 70u\mathbf{i} + 50(v\mathbf{i} + w\mathbf{j}).$$

$$70u + 50v = 420, \qquad 50w = 300 \Rightarrow w = 6.$$

Their speeds are equal, so Amos's speed squared equals Beth's speed squared. This supplies the second equation needed to determine u.

$$u^2 = v^2 + 6^2, \qquad v = \frac{420 - 70u}{50}.$$

$$u^2 = \left(\frac{420 - 70u}{50}\right)^2 + 36 \Rightarrow 2400u^2 - 58800u + 266400 = 0.$$

$$2u^2 - 49u + 222 = 0 \Rightarrow (u - 6)(2u - 37) = 0.$$

$$u = 6 \quad \textit{or} \quad u = 18.5.$$

Both values are positive, so both satisfy Amos travelling in the positive $\mathbf{i}$ direction, and each is the common speed.

The possible speed of each skater is 6.0 $\mathrm{m\,s^{-1}}$ or 18.5 $\mathrm{m\,s^{-1}}$.

(6)

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