Maths with Matt

Physics and maths resources

Edexcel OCR MEI A Level Further Mathematics

H645/Y434 - Numerical Methods

Wednesday 18 June 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

The values are $f(1.9) = 0.21692$, $f(2.0) = 0.20000$ and $f(2.1) = 0.18484$. Estimate $f'(2.0)$ using (a) a forward difference and (b) a central difference.

Model solution

(a) A forward difference uses the value at the point and the next value to its right. Here the step size is $h = 0.1$.

$$f'(2.0) \approx \frac{f(2.0+h) - f(2.0)}{h} = \frac{f(2.1) - f(2.0)}{0.1} = \frac{0.18484 - 0.20000}{0.1} = -0.1516.$$

$$f'(2.0) \approx -0.1516.$$

(2)

(b) A central difference is appropriate when values equally spaced on both sides of the required point are available. It normally gives a more balanced estimate.

$$f'(2.0) \approx \frac{f(2.1) - f(1.9)}{2(0.1)} = \frac{0.18484 - 0.21692}{0.2} = -0.1604.$$

$$f'(2.0) \approx -0.1604.$$

(2)

Question 2

Question 2

$P = 323$ is p rounded to the nearest whole number, and $Q = 162$ is q chopped to the nearest whole number. (a) Find the maximum possible relative error in using P for p, and in using Q for q. (b) Find the range of $R = 200/(p - 2q)$. (c) Explain why this range is so large.

Model solution

(a)(i) Rounding to the nearest integer gives an absolute error of at most 0.5. For the largest possible relative error, divide by the smallest possible true value, 322.5.

$$\frac{0.5}{322.5} = \frac{1}{645} = 0.001550\ldots$$

$$\textit{Maximum relative error} = \frac{1}{645} \approx 0.0016.$$

(1)

(a)(ii) Chopping q to 162 means $162 \leqslant q < 163$, so the error $Q - q$ can approach -1. Its greatest magnitude relative to q occurs as q approaches 163.

$$\frac{-1}{163} = -0.006134\ldots$$

$$\textit{Maximum possible relative error} \approx -0.0061.$$

(1)

(b) First translate each approximation rule into bounds, then use the fact that $p - 2q$ is negative throughout.

$$322.5 \leqslant p < 323.5, \qquad 162 \leqslant q < 163.$$

The denominator has bounds

$$322.5 - 2(163) < p - 2q < 323.5 - 2(162),$$

so

$$-3.5 < p - 2q < -0.5.$$

Since $200/d$ reverses its order as the negative denominator d increases,

$$\frac{200}{-0.5} < R < \frac{200}{-3.5}.$$

$$-400 < R < -\frac{400}{7} \textit{ (equivalently, with endpoint conventions, } -400 \leqslant R < -400/7\textit{).}$$

(3)

(c) The calculation divides by $p - 2q$, and $p \approx 2q$, so this denominator is close to zero. Division by a small number is unstable: small input changes produce large changes in $\mathbb{R}$.

The near-zero denominator $p - 2q$ magnifies the approximation errors.

(1)

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Question 3

Question 3

For $\int_{0.5}^{1.3} \sqrt{1+x^3}\,dx$, complete the table of midpoint, trapezium and Simpson estimates for $n = 1, 2$, then state the most accurate estimate justified by the table.

Model solution

(a) The table compares rules with increasingly fine subdivisions. Simpson's rule with $2n$ strips is listed alongside the midpoint and trapezium rules with n strips. For $n = 1$, the strip width is $h = 1.3 - 0.5 = 0.8$, so, for example,

$$M_1 = 0.8\sqrt{1+(0.9)^3} = 1.051932, \qquad T_1 = \frac{0.8}{2}\left(\sqrt{1+(0.5)^3} + \sqrt{1+(1.3)^3}\right) = 1.139470.$$

Using the same midpoint and trapezium formulae with $h = 0.4$, and Simpson's formula with 2 and 4 strips, gives the completed table:

n	M_n	T_n	S_{2n}
1	1.051932	1.139470	1.081111
2	1.074256	1.095701	1.081404

Completed entries: $M_1 = 1.051932$, $T_1 = 1.139470$, $S_2 = 1.081111$, $S_4 = 1.081404$.

(4)

(b) The two Simpson estimates agree through three decimal places, and their fourth decimal place is also 4 after rounding. This agreement is the evidence for the quoted precision.

$$\int_{0.5}^{1.3} \sqrt{1+x^3}\,dx \approx 1.0814, \text{ probably correct to 4 decimal places.}$$

(1)

Question 4

Question 4

Use Newton–Raphson, starting from $x_0 = 1$, to find the positive root of $\tanh x - x^2 + 4 = 0$. Show the first graphical construction, find x_1, x_2, and continue to give the root to 7 decimal places.

Model solution

(a) Newton--Raphson replaces the curve locally by its tangent. On the given graph, draw the tangent at $(1, f(1))$; where that tangent meets the x-axis is x_1, between 3 and 4.

The tangent intercept is labelled x_1.

(1)

(b) Let $f(x) = \tanh x - x^2 + 4$. Newton--Raphson is suitable because a differentiable formula for the curve is given, so we can repeatedly use its tangent intercept.

$$f'(x) = \operatorname{sech}^2 x - 2x, \qquad x_{n+1} = x_n - \frac{\tanh x_n - x_n^2 + 4}{\operatorname{sech}^2 x_n - 2x_n}.$$

Using $x_0 = 1$,

$$x_1 = 1 - \frac{\tanh 1 - 1^2 + 4}{\operatorname{sech}^2 1 - 2} = 3.38071713,$$

and substituting this new value back into the same iteration gives

$$x_2 = 2.42885430.$$

$$x_1 = 3.38071713, \quad x_2 = 2.42885430.$$

(4)

(c) Continue until consecutive iterates agree to at least eight decimal places; that supports a root quoted to seven decimal places.

n	x_n
3	2.23935179081
4	2.23097666893
5	2.23096009555
6	2.23096009549

The final two values have the same first eight decimal places.

Positive root $= 2.2309601$ (7 d.p.).

(2)

Question 5

Question 5

The largest positive root α of $f(x) = \sinh x - 3x + 1 = 0$ is found by the secant method from $x_0 = 2$, $x_1 = 3$. A spreadsheet gives $x_7 = 2.627264$ and the convergence-analysis columns shown in the question. Answer parts (a)–(f).

Model solution

(a) The secant method finds the x-intercept of the straight line through two known points $(x_r, f(x_r))$ and $(x_{r+1}, f(x_{r+1}))$. Thus the next value is

$$x_{r+2} = \frac{x_r f(x_{r+1}) - x_{r+1} f(x_r)}{f(x_{r+1}) - f(x_r)}.$$

For cell D4, which uses row 3, a suitable spreadsheet formula is

$$\boxed{= (B3 * E3 - C3 * D3)/(E3 - C3)}.$$

A suitable cell-D4 formula is $= (B3 * E3 - C3 * D3)/(E3 - C3)$.

(2)

(b) The latest spreadsheet iterate is the best available approximation.

$$\alpha \approx 2.627264.$$

(1)

(c) To test six-decimal-place accuracy, test points either side of the rounding boundary 2.627264: 2.6272635 and 2.6272645. A sign change proves a root lies between them.

$$f(2.6272635) \approx -0.00000037262, \qquad f(2.6272645) \approx 0.00000358.$$

These have opposite signs, so the root lies in $(2.6272635, 2.6272645)$, all of which rounds to 2.627264 to 6 d.p.

Yes, 2.627264 is accurate to 6 decimal places.

(2)

(d) If the second starting value were changed from 3 to 1, the two starting points would bracket the smaller positive root rather than the largest one.

The iteration would converge to the other (smaller) root.

(1)

(e) Column D is $\dfrac{x_{r+2} - x_{r+1}}{x_{r+1} - x_r}$, the ratio of successive changes. For first-order convergence this ratio would tend to a non-zero constant. Its magnitudes decrease, so the convergence is faster than first order.

The order of convergence is higher than first order.

(2)

(f) *Column E is* $\frac{x_{r+2} - x_{r+1}}{(x_{r+1} - x_r)^2}$. *This would settle to a constant for second-order convergence. Its magnitudes increase rather than settling, so the convergence cannot be second order and is slower than second order.*

The order of convergence is lower than second order.

(2)

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Question 6

Question 6

Midpoint-rule estimates M_n for $\int_0^1 \sqrt{\sin x}\,dx$ are supplied, with differences and successive-difference ratios tending to about 0.358. Explain the order and obtain the best estimate possible from the table.

Model solution

(a) When n is doubled, an order-p error is multiplied approximately by 2^{-p}. The ratios tend to 0.358, which lies strictly between $0.25 = 2^{-2}$ and $0.5 = 2^{-1}$.

The order is between first and second order in this case.

(2)

(b) Use the limiting ratio to extrapolate away the remaining geometric error. With $M_{128} = 0.6430189$, latest difference $d = -7.463 \times 10^{-5}$, and $r \approx 0.35801$, the limiting value is

$$I \approx M_{128} + d\frac{r}{1-r} = 0.6430189 + (-7.463 \times 10^{-5})\frac{0.35801}{1-0.35801} = 0.64297728\ldots$$

The extrapolation is more accurate than the raw midpoint estimate because it estimates and removes the leading remaining error.

$$\int_0^1 \sqrt{\sin x}\,dx \approx 0.64298$$ *(so 0.6430 is secure to 4 d.p.).*

(5)

Question 7

Question 7

For $f(x) = 8\ln(x+2) - 3x + 1 = 0$, use $g(x) = 8\ln(x+2) - 2x + 1$ and the stated relaxed iterations to discuss and find its positive root γ and negative root δ.

Model solution

(a) For a fixed-point iteration, the size of $|g'(\gamma)|$ controls the linear convergence rate. Here $|g'(\gamma)| = 0.977$, very close to 1, so each error is only reduced slightly at each iteration.

Convergence will be very slow.

(2)

(b) The best evidence is agreement of the two latest displayed iterates: $x_3 = x_4 = 5.8166999$.

$\gamma = 5.8166999$, to the displayed precision.

(1)

(c) Spreadsheet cells store more digits than they display. The stored values in F5 and F6 are different but both round for display to 5.8166999. Cell G uses those full stored values to calculate $f(x)$, hence its two displayed values can be different.

The apparent equality in F is due to display rounding; G uses unequal stored values.

(2)

(d) An unrelaxed fixed-point iteration is locally convergent only when $|g'(\delta)| < 1$. Here $g'(\delta) = 13.9 > 1$.

It is divergent near δ, so it may not be used to find δ.

(1)

(e)(i) Relaxation replaces $x_{n+1} = g(x_n)$ by a weighted average of x_n and $g(x_n)$:

$$x_{n+1} = (1 - 0.506)x_n + 0.506g(x_n), \qquad x_0 = -1.$$

Successive substitution gives

r	1	2	3	4
x_r	1.024	4.455	5.747	5.817

$x_1 = 1.024,\ x_2 = 4.455,\ x_3 = 5.747,\ x_4 = 5.817.$

(2)

(e)(ii) These iterates have crossed to the attraction region of the positive root and are approaching 5.8166999.

The iteration converges to γ, the positive root.

(1)

(f) *With negative relaxation, use* $m = -0.078$ *so that the iteration is stable near the negative root:* $x_{n+1} = (1-m)x_n + mg(x_n)$. *Starting at* -1,

n	1	2	3	4	5	6
x_n	-1.312	-1.463652941	-1.495412060	-1.496514281	-1.496509866	-1.496509890

The final two values agree to eight decimal places.

$$\delta = -1.4965099 \text{ (7 d.p.).}$$

(2)

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Question 8

Question 8

Times t for a 100 m run are $(W, t) = (0, 18.00), (2, 17.68), (3, 16.83)$, later with $(4, 15.44)$. Construct the requested interpolation models and use the newer model to assess qualification at $W = 6$.

Model solution

(a) There are three data points, so the Lagrange polynomial of degree 2 is appropriate. Each basis term is 1 at its own week and 0 at the other two weeks, ensuring the model passes through every given point.

$$t = 18\frac{(W-2)(W-3)}{(0-2)(0-3)} + 17.68\frac{W(W-3)}{(2-0)(2-3)} + 16.83\frac{W(W-2)}{(3-0)(3-2)}.$$

Expanding and collecting powers of W gives

$$t = -0.23W^2 + 0.3W + 18.$$

$$t = -0.23W^2 + 0.3W + 18.$$

(3)

(b) Test the model at the new observed week, $W = 4$, and compare it with the observed 15.44 seconds.

$$t = -0.23(4)^2 + 0.3(4) + 18 = 15.52.$$

This is close to 15.44.

Yes; the model is a good fit for these values.

(1)

(c) The weeks $2, 3, 4$ are equally spaced, so Newton's forward difference formula can use the first differences $-0.85, -1.39$ and second difference -0.54, starting at $W = 2$:

$$t = 17.68 - 0.85(W-2) - \frac{0.54}{2!}(W-2)(W-3).$$

Expanding,

$$t = -0.27W^2 + 0.50W + 17.76.$$

Comparing with $t = -0.27W^2 + BW + C$,

$$B = 0.5, \quad C = 17.76.$$

(3)

(d) A model intended to predict the athlete's current progress should favour recent performance.

It is a refinement because it uses the three most recent data points.

(1)

(e) Substitute the competition week into the newer model and compare with the strict requirement $t < 11$.

$$t = -0.27(6)^2 + 0.5(6) + 17.76 = 11.04 \text{ s}.$$

No. The model predicts 11.04 *s* > 11 *s, so the athlete will not qualify.*

(1)

(f) This would extrapolate beyond the data, but the more important issue is that the quadratic predicts an unrealistic continuing fall in time. For example, it gives $t = 8.03$ *s at* $W = 7$*, far below a plausible 100 m time, and eventually predicts negative times.*

No; the model gives physically implausible times beyond this range.

(1)

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